

the eastern Highland Rim, and finally, to the discovery of the factors responsible for the recent greatly increased populations of both species of Compositae.

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**SEMI-EMPIRICAL INVESTIGATIONS  
ON  
THE NATURE OF THE f-VALUE<sup>1</sup>**

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## ABSTRACT

It is suggested that an analysis of available f-value for atoms and ions would yield semi-empirical understanding of the emission process and would be temporarily useful in technological applications. Current literature search for this purpose is reviewed. A sample analysis, carried out with calculated transition probabilities of Pasternack (1940) is described. Promising experimental studies are suggested.

## INTRODUCTION

In the early days when oscillator strengths of all the lines of any element, measured with canal ray experiments, were thought to be the same, Kerschbaum (1927) suggested comparisons of the various resulting values in a search for understanding of atomic radiation. Vastly improved measurements and well-advanced calculations of f-values for neutral and ionized atoms totaling hundreds of lines suggest that a renewed effort be made to study the emission process by comparing the available figures.

A second motivating cause for such a comparison is the fact that, though many f-values are indeed available, there are not nearly enough to meet the diverse needs of a rapidly expanding technology. Measurements of high temperatures in all sorts of new devices require new f-values, though not of very great pre-

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cision. It appears that interpolation of unknown *f*-values for a given line from those known *f*-values of similar lines of other atoms and ions might help fill needs in given situations. Here again the first step would be a study of available figures.

### PROCEDURES

A literature search and compilation was begun at Southern Missionary College but was soon absorbed by current work at the National Bureau of Standards. The Atomic Physics Division (Code 13) has established a "Data Center" which will eventually interest itself in the collection, evaluation, and publication in useful form of many atomic data (NBS Report, 1959). The *f*-value collection will concern itself only with numbers of fair precision, say  $\pm 50\%$  (absolute *f*) or better. It is not yet established whether the collection will include such high-precision intensity measurements as those of Crosswhite (1950) or of Heid and Dieke (1954), or the many calculations of transition probabilities for ions for which few corroborative experimental values exist. This program will undoubtedly occupy several years. Another program of the Bureau, under the direction of Corliss, Meggers, and Scribner (report to be published), is the measurement of approximate (factor of 2) intensities for 38,000 lines of some 70 elements. Completion of the 25-year project is anticipated this year. These two NBS programs promise to make a wealth of information available on lines of neutral atoms and singly ionized atoms.

The following types of comparisons of data are in progress at this laboratory:

1. "Vertical" samples down the periodic table of neutral atoms (H I, Li I, Na I, . . . ; He I, Ne I, Kr I, . . .). See for instance Bethe.
2. Comparisons of alkalis and alkali metals with halogens and the oxygen-like atoms (one or two valence electrons compared with a shell lacking one or two electrons of being closed) (Ufford, 1935).
3. Lifetimes have been measured under a variety of conditions (e.g. Osherovich and Savich, 1958; Cojan and Thibeau, 1959). It would be of considerable interest to investigate changes in the lifetime or oscillator strength as the atom was immersed in more and more dense media.<sup>1</sup>
4. Isoelectronic sequences. At the present time very little experimental work has been possible on other than neutral and singly ionized atoms. (See footnote on previous page.) Even for these it is rare to find a case where a given line has been measured in both species (e.g. the 2s-2p transition in Li I and Be II).

We recently carried out a short pilot study with the calculated transition probabilities of Pasternack (1940), who considered lines of several isoelectronic sequences. It was recognized at the outset that calculated values often exhibit large deviations from experiment, that the conclusions (if any) would apply only

<sup>1</sup>Some measurements of triply ionized atoms in liquids and crystals exist (Landolt-Bornstein, 1950).

to the electric quadrupole and magnetic dipole transitions he calculated, and that there was no hope of "discovering" anything not already present in the theory which yielded the calculated values. A description of this study follows.

### CALCULATIONS

For hydrogen-like ions the  $f$ -value for a given line is a constant (Allen, 1955); from this it follows that  $A$ , the transition probability, is proportional to  $Z^4$ , cf. Condon and shortley (1957) and Unsold (1955). This relation is thought to hold even for such extreme systems as mu-mesonic atoms (Stearns and Stearns, 1957, and Bernstein and Wu, 1959). For other sequences one might expect (White, 1934), to a first approximation, that  $A$  would be proportional to  $Z^{*4}$ , where  $Z^*$  is some effective nuclear charge (total minus shielding); thus

$$A = k(Z-s)^4 = kZ^{*4}.$$

By graphing, for a given transition,  $A^{1/4}$  against  $Z$ , it would be possible to infer  $k$  and  $s$ .<sup>2</sup>

The information contained in several randomly-selected rows of tables 7a to 7f of Pasternack's work was plotted in this manner. It became evident that some of the "curves" thus obtained were linear and some appeared parabolic or cubic. In order to determine the nature of the dependence upon  $Z$ , the quantities  $A^{1/4}$ , and their first and second differences were calculated for each line given by Pasternack. The averages of these quantities,  $\bar{d}$  and  $\bar{d}^2$ , were computed and their probable errors estimated. These calculations indicated the existence of four situations which were treated as follows:

1.  $\bar{d}$  "constant" (varies irregularly about the mean; the probable error of  $\bar{d}^2$  will exceed that of  $\bar{d}$ ).  $A^{1/4}$  was plotted against  $Z$ , and  $k$  and  $s$  were read off the graph.
2.  $\bar{d}$  showed a trend (each  $A^{1/4}$  larger (smaller) than that for preceding  $Z$ —or if just one was slightly too large or too small in be in sequence),  $\bar{d}^2$  "constant", and  $Z$  between 6 and 16 (Tables 7a, 7c, and 7e). Here a parabolic fit was made, as follows. It was assumed that

$$A^{1/4} = a + bZ + cZ^2.$$

The best or averaged value of  $c$ ,  $\bar{c}$ , is just  $\bar{d}^2/2$ . One can then write as many equations as there are known values of  $A^{1/4}$ , and with just *two* unknowns,  $a$  and  $b$ . This allows solution for several values both  $a$  and  $b$  (for this one sequence) which can be averaged. From  $\bar{a}$ ,  $\bar{b}$ , and  $\bar{c}$  it was possible to solve for the possibly more physically

<sup>2</sup>The calculation of the energy levels, between two of which a transition must take place—and which therefore is a simpler problem than that of the transition probability—is in itself a major undertaking. The variation of levels in isoelectronic sequences has been considered by Layzer (1959) and by Silverman and Matsen (1960).

significant parameters  $g$ ,  $h$ , and  $s$  in another form of the parabolic equation,

$$A^{3/4} = g + h(Z - s)^2.$$

3. Same as #2 except  $Z$  between 14 and 26. Here the simultaneous solutions became too insensitive to the magnitude of  $A^{3/4}$  due to the size of  $Z^2$ . A linear fit as in #1 would be indicated but has not yet been done.
4.  $d^2$  also shows a trend—cubic or higher power curve. Not studied.

It is evident that nothing is to be gained from this pilot study which was not implicit in Pasternack's assumptions. It will not be surprising, then, that a correlation was found between the effective number of shielding electrons ( $s$ ) and the angular momentum quantum number of the lower level, for those isoelectronic sequences for which  $A^{3/4}$  is linear against  $Z$ . It is, perhaps, surprising that no more correlations were evident.

### CONCLUSIONS

It would be highly desirable if experimental  $f$ -values were available for highly-ionized atoms. In addition, it is to be observed that a given line in the visible for, say Li I, will be far in the very soft X-rays for Ca XVIII; conversely a visible Ca XVIII line would be for Li I in the radiofrequency spectrum. Search in the literature for transition probability measurements in other parts of the spectrum than the visible (and nearby) regions has indicated that very little work has been done. Such experiments would appear very fruitful and would, at least in the case of radiofrequency emissions of neutral atoms, appear possible.

Ultimately all such studies with experimental data may be hoped to aid the understanding of oscillator strengths, much as the Ritz combination principle and Balmer's series helped in the formulation of present atomic theory. This would be particularly true if enough data of high precision were available to yield curves where real irregularities from the expected "smooth curves" were to be seen. (See for instance Filipov (1931), fig. 6.)

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