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SUMMARY OF GROUND-WATER LEVELS IN TENNESSEE, 1952-61[1]

ROY H. BINGHAM
Engineering Technician, U. S. Geological Survey
Nashville, Tennessee

ABSTRACT

In areas not affected by ground-water pumpage, water levels in Tennessee fluctuate from less than a foot to slightly more than 15 ft. between yearly highs and lows. These changes are caused almost entirely by variations in the amount of water entering the ground-water reservoirs from precipitation. The variations in the amount of water entering the ground-water reservoirs are, in turn, regulated by the portion of the precipitation that is intercepted by evaporation and transpiration. Long-term trends and the yearly net rise or decline of water levels in areas not influenced by pumpage are caused by annual variations in the amount of precipitation.

The effect of pumpage on water levels in nearby wells depends on several factors, including the rate of pumping, the permeability and storage characteristics of the aquifer, and the distance of the observation well from pumping wells. The relationship of these factors to water levels in typical wells is discussed.

Short-term and seasonal variations in water-level fluctuations are strongly influenced by local geologic and topographic conditions. Seasonal fluctuations of water level in unconsolidated sands and gravel in the rural areas of west Tennessee are generally less than 2 ft. In the limestone terrane of central Tennessee, however, the seasonal range of water-level fluctuations is from about a foot to sightly more than 15 ft. unlike seasonal or short-term fluctuations the long-term trends in water level have little relationship to geology and geography, although the magnitude of water-level changes may be related indirectly to these factors.

INTRODUCTION

The collection of water-level data in Tennessee is part of a continuing and systematic program of the U. S. Geological Survey to evaluate and describe the ground-water resources of the nation. Water-level data may be used as an index to ground-water availability, to explain rising or falling water levels, and to predict future water-level trends.

Water levels in Tennessee are not stationary; they fluctuate in response to many different factors such as changes in precipitation, variations in the rate of evaporation and transpiration, changes in barometric pressure, earth tides, earthquakes, loading of the land surface, and pumpage from wells (Meinzer, 1923, WSP 489, p. 31). These factors vary in magnitude and time and thus produce an irregular record of water levels that are sometimes complicated to interpret. The

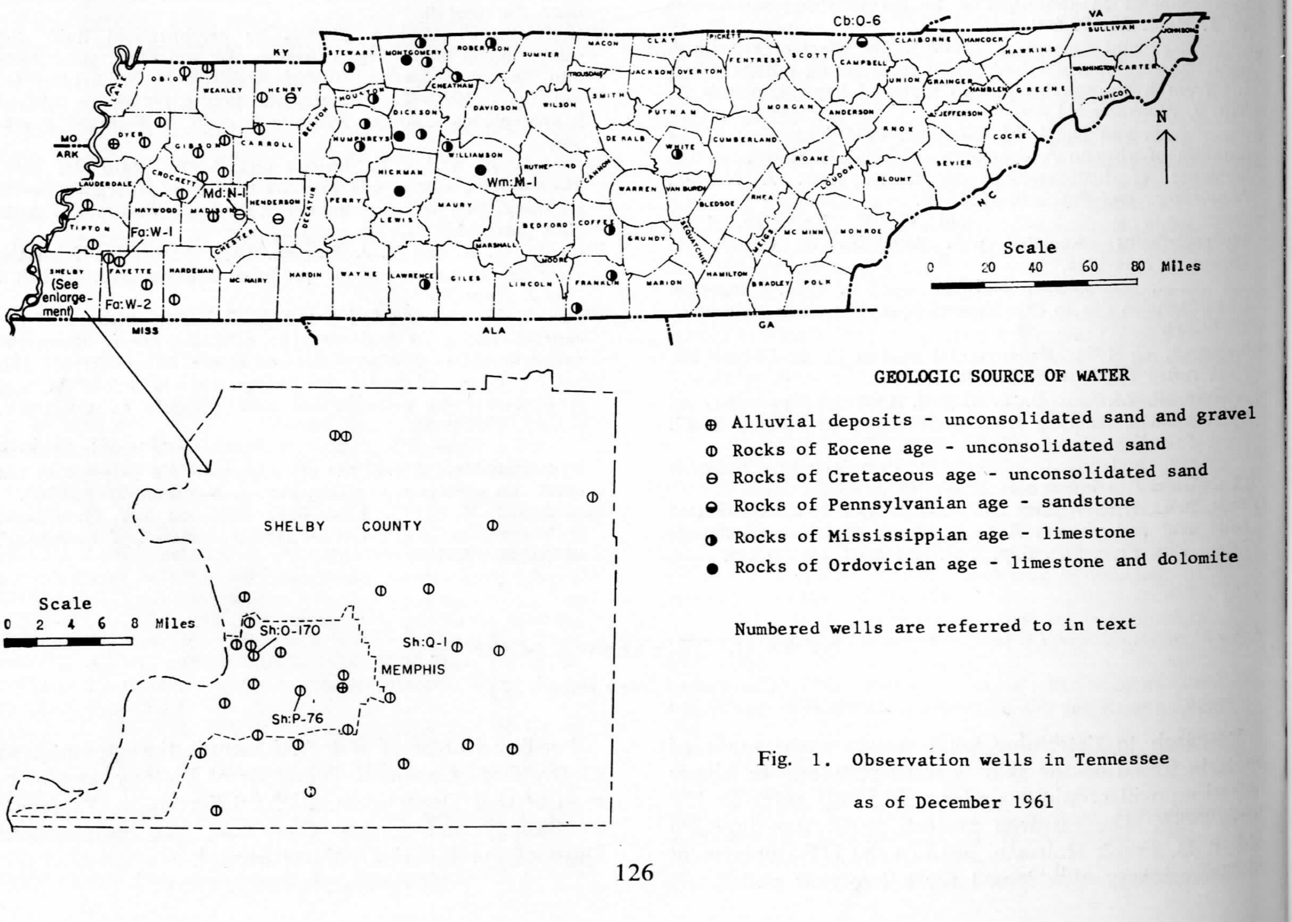

Fig. 1. Observation wells in Tennessee as of December 1961

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[1] Work done in cooperation with Tennessee Division of Water Resources.

purpose of this report is to describe the nature of water-level fluctuations that are caused by changes in precipitation, variations in the rate of evaporation and transpiration, and pumping, because these factors have the largest and most important effects upon water levels.

The U. S. Geological Survey, as of December 1961, maintained 71 observation wells in Tennessee. The location of the observation wells and geologic source of water in each well is shown in Fig. 1. Twenty-nine of the wells are located in Shelby County and are maintained in cooperation with the city of Memphis. The remaining wells, including 22 in west Tennessee, 19 in central Tennessee, and 1 in east Tennessee are maintained in cooperation with the Tennessee Division of Water Resources. Additional observation wells should be added to the network, mainly in central and east Tennessee, to overcome the deficiency of basic water-level information in those areas.

FLUCTUATIONS OF WATER LEVELS

Water levels in Tennessee generally are cyclical from season to season. The highest water levels occur during the rainy season and low levels occur during the dry season. The cyclic fluctuations reflect seasonal variations in the rate of recharge to and discharge from the ground-water reservoirs or aquifers.

The relation between the amount of precipitation and the amount of recharge is influenced strongly by the rate of evaporation and transpiration. The rate of evaporation and transpiration is lowest during the winter rainy season because of cooler temperatures and the dormancy of vegetation, and highest during the summer dry season because of warm temperatures and active plant growth. A large amount of precipitation is intercepted by evaporation and transpiration during the dry summer months. As a result the amount of water that enters the aquifer in the summer is reduced to nothing or almost nothing. In fact, shallow aquifers may supply much of the water used by vegetation during drought conditions. These losses cause additional declines in water level.

Long-term trends of water levels in urban areas of west Tennessee are generally downward, largely as a result of increases in pumpage. Superimposed on this downward trend are seasonal rises and declines resulting from variations in the amount of water in storage in ground-water reservoirs. In rural areas of Tennessee long-term trends of water levels seemingly are influenced by annual variations in the amount of precipitation.

Effects of pumpage in consolidated and unconsolidated sand aquifers

The hydrographs (graphic plots of water level versus time) of an observation well at Jellico (Cb:O-6) and a well in Memphis (Sh:P-76) provide a comparison of the local effects of heavy pumpage on consolidated and unconsolidated sand aquifers (Fig. 2). The hydrograph for well Cb:O-6 was plotted from lowest and highest monthly water-level measurements. Well Cb:O-6 was drilled into cemented sandstone of Pennsylvanian age at Jellico, Tennessee. Pumpage from this aquifer in the vicinity of well Cb:O-6 causes some changes in the water level in the observation well, but the amount of pumpage and the resultant change in water level is small and temporary. The major water-level fluctuations in this well are caused by variations in the amount of water stored in the aquifer. The peaks in the dashed curve, or high-water levels, coincide with "highs" in precipitation, and the lowest points on the curve coincide with "lows" in precipitation.

The lowest monthly water level, or solid curve, reflects the long-term trend in water level. As shown by this curve, there is no overall decline or rise of water level for the period 1952 through 1961. This curve also shows the seasonal effects of variations in the rates of evaporation and transpiration as the water level declined during the dry periods and began to rise at the onset of the rainy seasons. The seasonal cycles are shown on the graphs of both wells in Fig. 2, although the highs and lows are accentuated by changes in pumping.

The usual seasonal cycle in water level of well Cb:O-6 (Fig. 2) changed during the period 1959 through 1961. A general rise in water level occurred during 1959, which probably was caused by a more equal distribution of precipitation throughout the year. During the latter part of 1960 the water level declined only about half as much as in preceding years. This temporary rise probably was caused by increased snowfall during February and March, which served as a barrier to runoff as the snow melted and evaporated slowly, thereby increasing the amount of water entering the aquifer (Meinzer, 1923, WSP 494, p. 35). In 1961 the rate of precipitation was about normal and the water level returned to normal in November.

The lower hydrograph (Fig. 2) was plotted from water-level records of a well (Sh: P-76) in the "500-foot" sand of the Claiborne Group in the Memphis area. This graph shows that changes in water level in this well, which is located near the center of pumping for the area, are the combined result of heavy pumpage and variations in the rate of recharge to the aquifer from precipitation (Criner, 1958, p. 16-17). The increase in pumpage from 1952 to 1961 resulted in a general downward trend in the water level which amounted to approximately 15 ft. during the 10-year period. Most of the decline occurred during 1953 and 1954 at a rate of approximately 5 ft. each year. During this period the amount of recharge was below normal and the amount of pumpage was above normal. The water level remained approximately the same during the next 2 years in spite of the fact that pumpage increased. The lack of decline reflects increased recharge to the aquifer.

Recharge to the aquifer probably increased in 1957 and pumpage decreased, which resulted in a rise in water level. The water level continued to rise in 1958 in spite of the fact that total pumpage increased and precipitation decreased. The rise probably was caused by a decrease in pumpage during the month of June when pumpage usually is very heavy.

The water level continued its general downward trend from 1959 through 1961 (Fig. 2). Pumpage again increased in 1959 and lowered the water level approximately 4 ft. The increased pumpage was primarily a

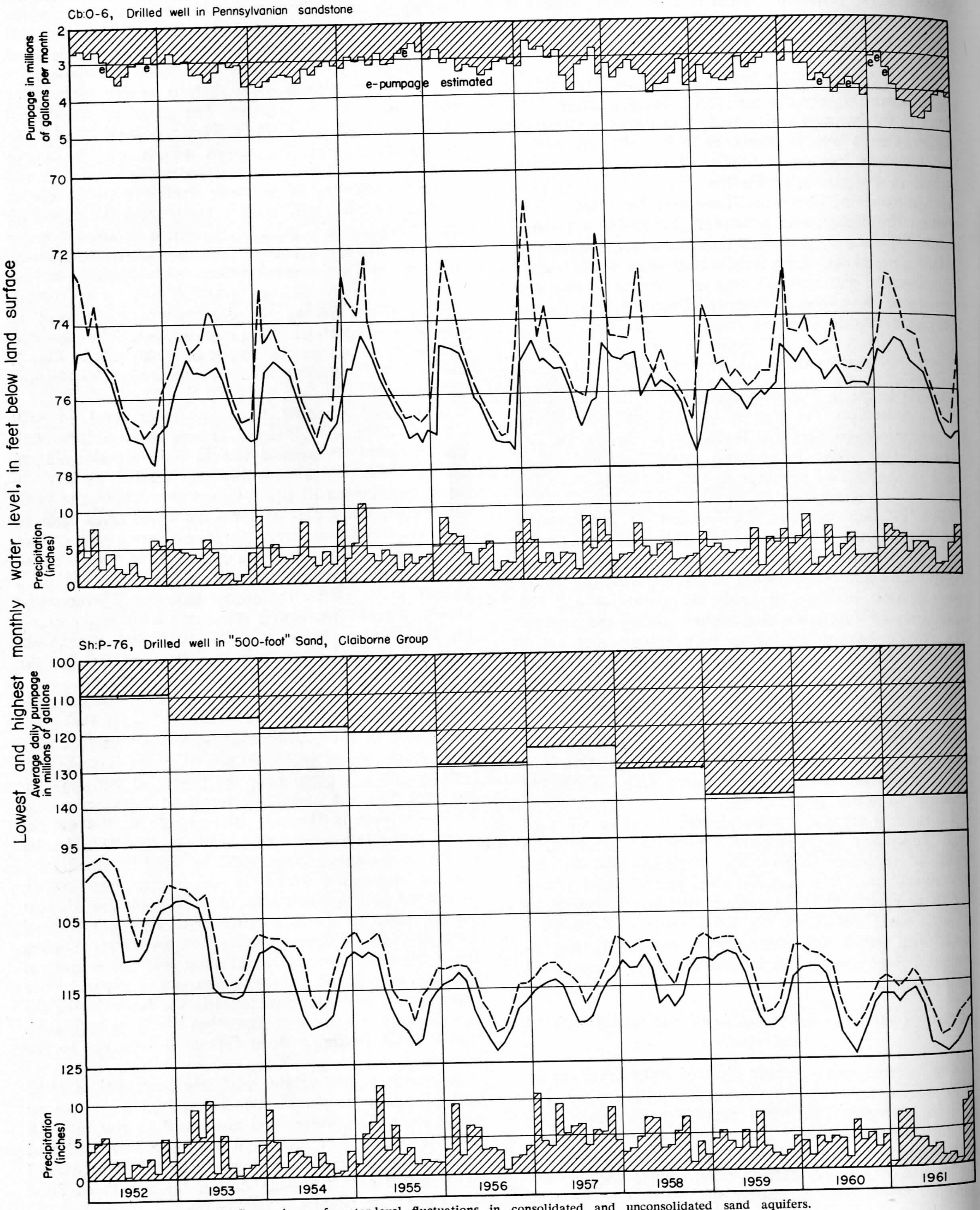

Fig. 2. Comparison of water-level fluctuations in consolidated and unconsolidated sand aquifers.

result of dry conditions during the year rather than additional development of new wells. Although pumpage decreased in 1960, the water level declined approximately 4 ft., probably as a result of a decrease of recharge into the aquifer.

Effects of heavy pumpage versus natural conditions

A comparison of the water-level fluctuations in an area of heavy pumpage with those in an area totally unaffected by pumpage is shown in Fig. 3.

The upper hydrograph shows water-level changes in a well (Sh:O-170) screened in the "1,400-foot" sand and located at the Parkway Pumping Station near the center of pumping at Memphis. Although the general trend in water level is downward, seasonal changes in water level do not correlate with the total pumpage as shown at the upper edge of the graph. This lack of correlation is due largely to periodic rotation of pumpage between the Parkway and other pumping stations. Pumpage effects, however, are pronounced during those periods when the Parkway station is in full operation.

The lower hydrograph (Fig. 3) is plotted from water-level records of a well (Wm:M-1) in the Knox Dolomite. The well is far removed from any significant pumpage, and the fluctuations of water level are the results of natural influences. The graph shows a reversal in the usual water-level cycles in that water levels are highest during the dry season and lowest during the wet season. The reversal is probably due to a lag in the time required for recharge to reach the well. The total

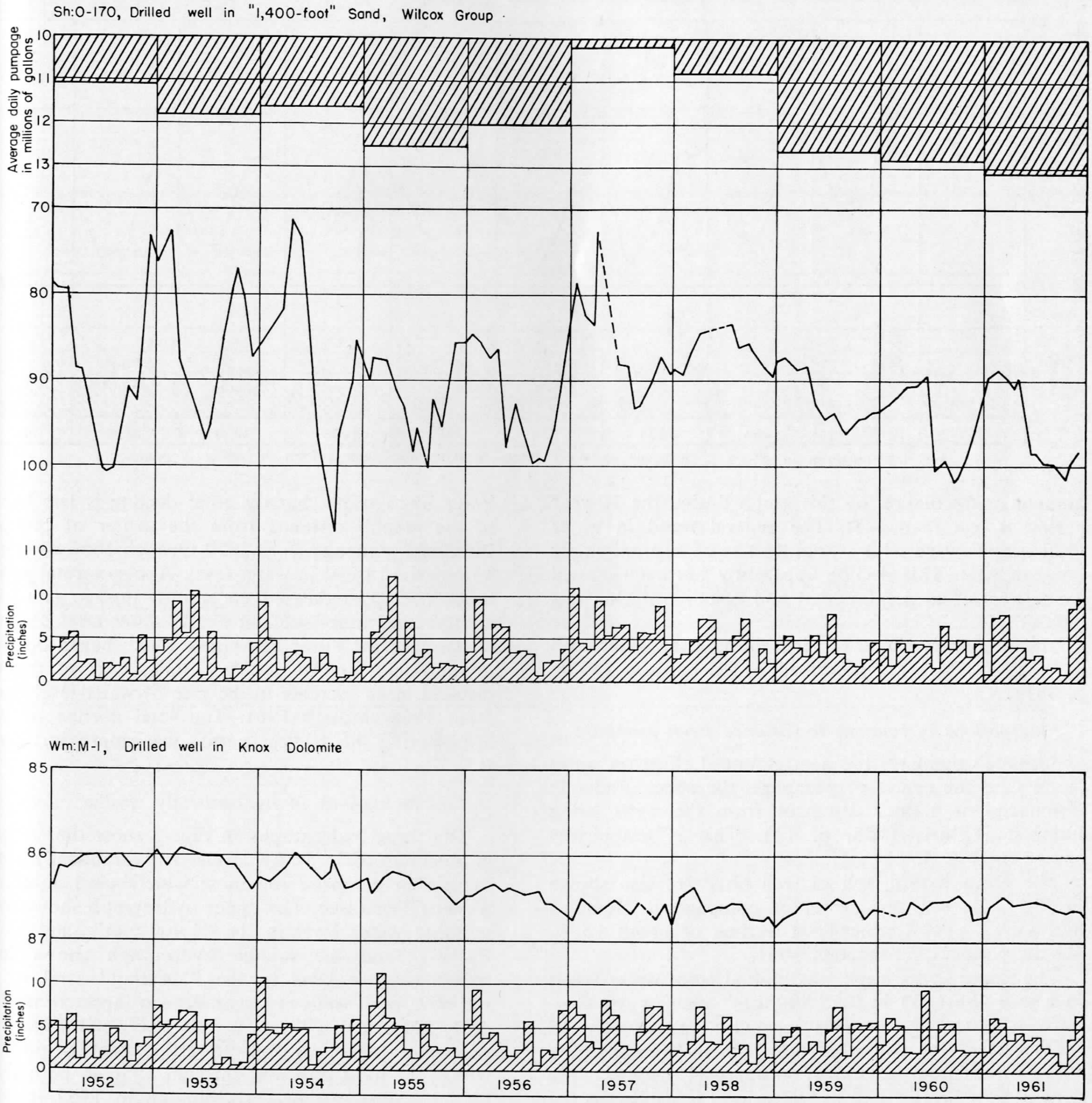

Fig. 3. Comparison of water-level fluctuations resulting from heavy pumpage and natural influences.

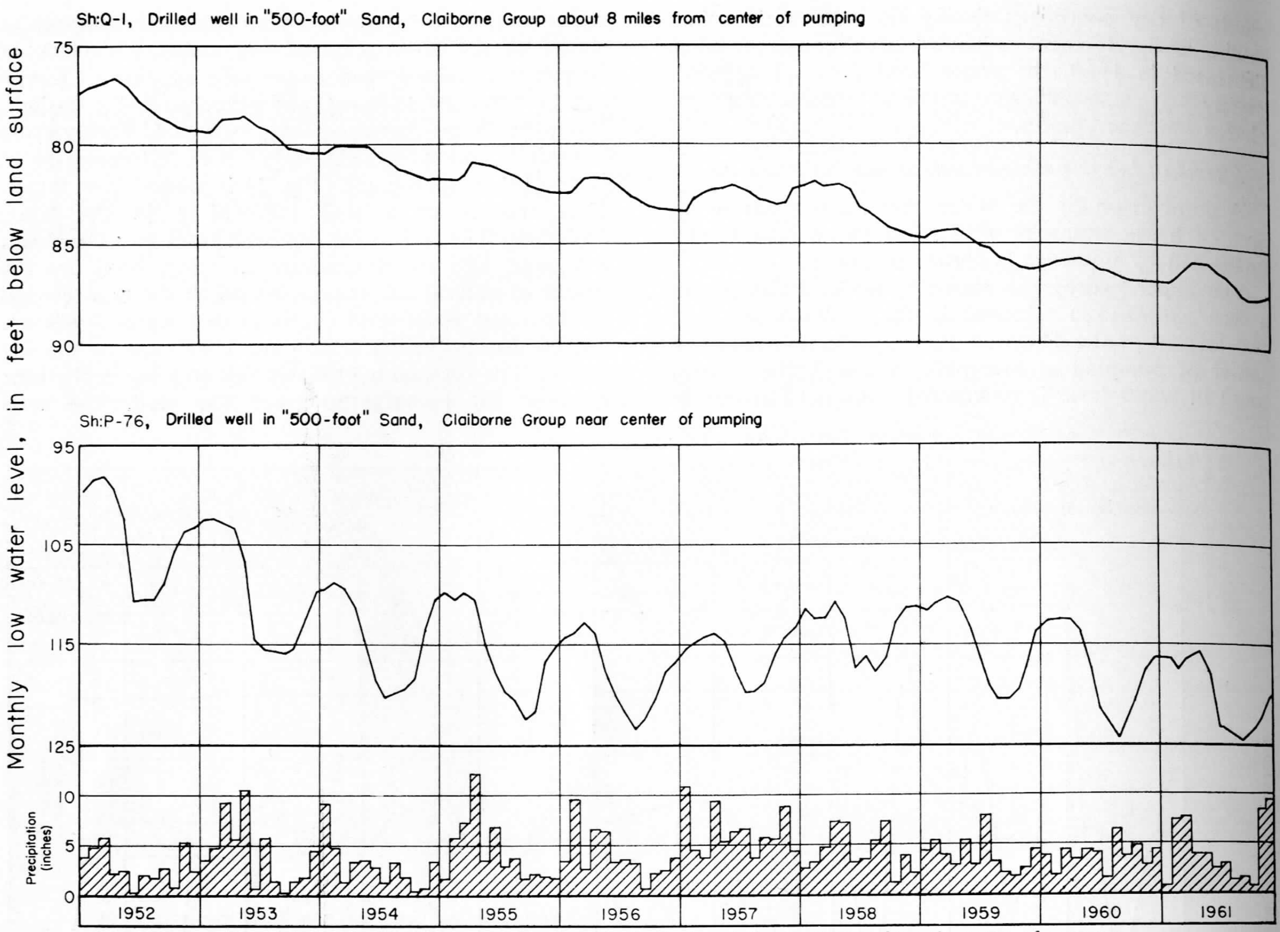

Fig. 4. Comparison of effects of pumpage on water levels in relation to distance from the center of pumpage.

amount of fluctuation on this graph during the 10-year period is less than 1 ft. The general trend in water level is downward with a total decline of approximately one-half foot. This decline apparently has been caused by below-normal precipitation and recharge during this period.

The temporary effects of earth tides on the water level in well Wm:M-1 are described by Richardson (1956, p. 461).

Fluctuations in relation to distance from pumpage

Although pumpage has a pronounced effect on water levels near the center of pumpage, the effect gradually diminishes at greater distances from the wells being pumped (Criner, 1958, p. 11). This is graphically shown on Fig. 4.

The lower hydrograph of well Sh:P-76, also shown in Fig. 2, is near the center of pumpage at Memphis and shows a total water-level decline of about 15 ft. for the period 1952 through 1961.

The upper hydrograph was plotted from water levels in a well (Sh:Q-1) in the "500-foot" sand of the Claiborne Group. The well is located approximately 10 miles east of the center of pumpage in Memphis and reflects the areal extent of water-level decline. The general downward trend in water level is similar to the lower hydrograph, but the total decline is less because of the greater distance from the center of pumpage. Increased pumpage from 1952 through 1956 resulted in a downward trend in water level. Above-normal precipitation in 1957 increased the rate of storage gain in the aquifer and caused a slight rise in water level. Pumpage at the McCord Pumping Station which began operation in mid-1958, approximately 3 miles west of the well, resulted in an increase in the rate of water-level decline from 1958 through 1961. The total decline of water level during the 10-year period was approximately 7.5 ft.

Fluctuations in hydraulically similar sands

The three hydrographs in Fig. 5 show the similarity of fluctuations in water levels in hydraulically similar sands that comprise the most widely used aquifers in western Tennessee. The upper hydrograph shows lowest monthly water level in the "1,400-foot" sand in well Fa:W-1, and the middle hydrograph shows lowest monthly water level in the "500-foot" sand in well Fa:W-2. Both wells are near Braden, approximately 30 miles northeast of the center of heavy pumpage in the Memphis area. Each well shows the effects of pumpage, which are most apparent through 1956 in both hydrographs. Above-normal precipitation in 1957 increased the amount of recharge into the aquifer, and a decrease in pumpage from the "1,400-foot" sand of about 1.8 mgd (million gallons per day) combined to cause the water level to rise in well Fa:W-1 during 1957 and 1958. A smaller rise would be expected from the small decrease in pumpage, and most of the rise during the 2-year period therefore is attributed to an increase in recharge to the aquifer. The water level declined approximately 2 ft. during 1959 through 1961 because of an increase in pumpage from the "1,400-foot" sand. The overall decline during the 10-year period was about 6 ft.

The middle hydrograph shows that the water level declined approximately 4 ft. from 1952 through 1956 in well Fa:W-2 because of an increase in pumpage from the "500-foot" sand in the Memphis area. Pumpage decreased about 4 mgd in 1957 and precipitation increased in the outcrop area, which resulted in an in-

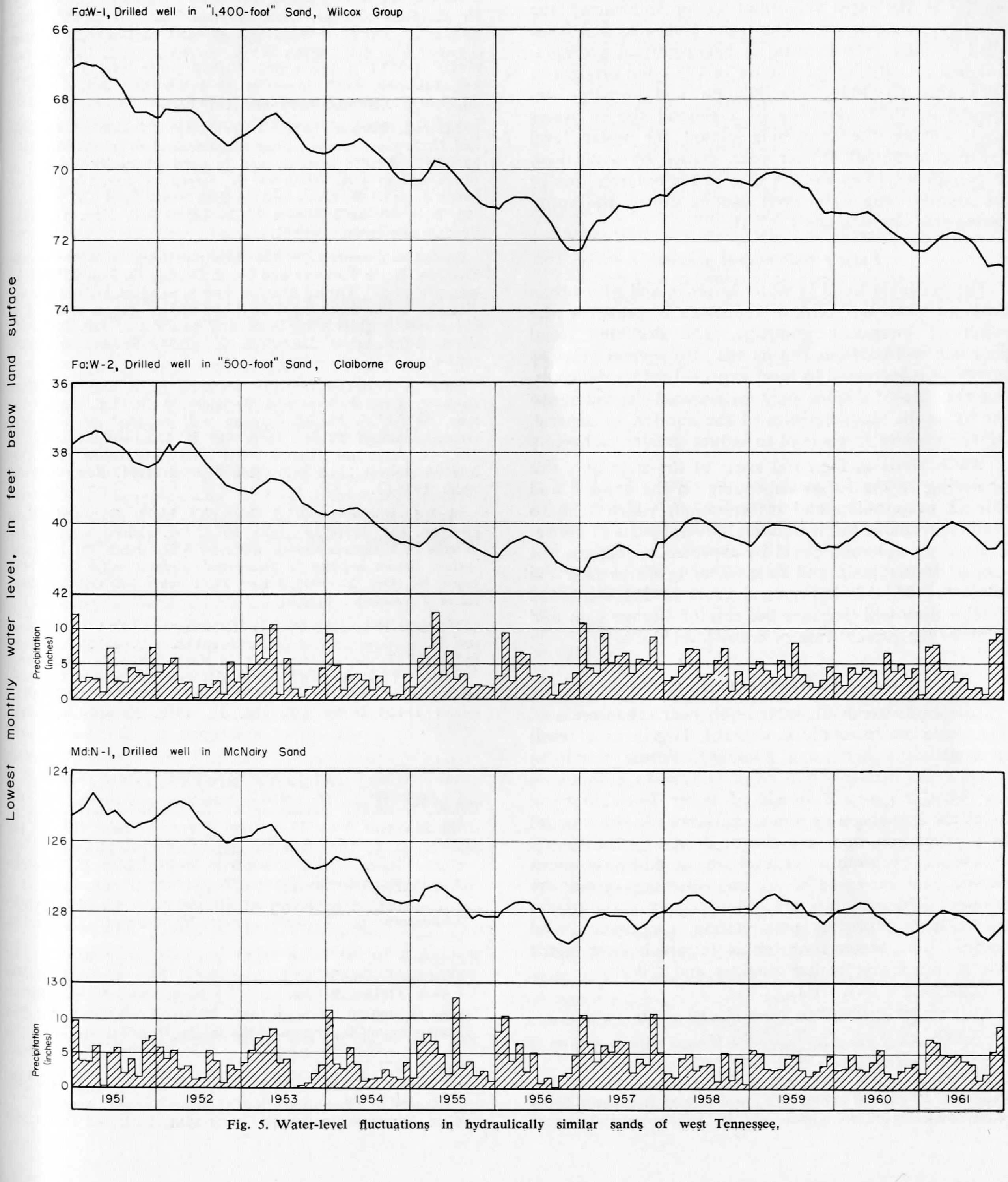

Fig. 5. Water-level fluctuations in hydraulically similar sands of west Tennessee.

crease in recharge to the aquifer and a rise in water level in 1957 and 1958. Most of the rise is attributed to an increase in the rate of storage gain in the aquifer. The water level remained rather constant from 1959 through 1961.

The lower hydrograph shows lowest monthly water level in McNairy Sand in a well (Md:N-1) at Claybrook, approximately halfway between Jackson and Lexington. Variation in the amount of storage in the aquifer is the most important factor influencing the water level in this well. The water level declined from 1952 through 1956 because of below-normal precipitation and recharge to the aquifer in 1952 and drought in 1953 through 1956. Precipitation and recharge increased in 1957, resulting in a general rise of water level. During the remaining period the water level declined about 0.1 ft. per year, caused by a decrease in precipitation and storage gain in the outcrop area of the aquifer. The water-level decline during the entire period was approximately 3.5 ft.

Future water-level trends

The declining trend in water levels in and near urban areas of west and central Tennessee is primarily the result of increasing pumpage. The declining trend probably will continue during the next several years as pumpage is increased to meet expected future demands, but the rate of decline may be reduced depending on the hydraulic characteristics of the aquifer. In general, declining water levels tend to induce greater recharge.

Water levels in the rural areas of the state may rise or decline in the future depending on the amount and rate of evaporation and transpiration, which tend to regulate recharge to the aquifers. Several years of above-average precipitation could be expected to increase the rate of storage gain and raise water levels to near the levels of 1951-52, but several years of below-average precipitation will decrease the rate of storage gain and increase the present rate of water-level decline.

SUMMARY

Long-term trends of water levels near urban areas of Tennessee are generally downward, largely as a result of continually increasing pumpage. Future trends in urban areas probably will be downward as pumping is increased. Long-term trends of water levels in rural areas are influenced by annual variations in the amount of precipitation, and may rise or decline in the future.

Seasonal cycles in water level are caused by variations in the rate of recharge to and discharge from the aquifers. These factors are influenced by variations in the rates of pumping, precipitation, evaporation, and transpiration. Water levels generally are highest in the winter and lowest in late summer and fall.

DESCRIPTION OF OBSERVATION WELLS

(Measurements for these wells and others are listed in Part 2 of Water-Supply Papers 1222, 1266, 1322, and 1405.)

Cb:O-6. Formerly 7:1-6. Jellico Water Works. Lat. 36°35′, long. 84°04′. Unused drilled well, probably in Rockcastle Sandstone, diameter 12 in., depth 620 ft. Land-surface datum is about 1,020 ft. above msl. Highest water level 70.81 below lsd, Feb. 2, 1950; lowest 87.04 below lsd, Aug. 10, 1949. Records available: 1949-61.

Fa:W-1. Formerly 24:10-1. Tennessee Division of Geology and U. S. Geol. Survey. Near Braden. Lat. 35°22′26″, long. 89°32′52″. Drilled artesian well in sand of Wilcox Group, diameter 6 to 4 in., depth 1,025 ft., screen 1,008-1,025. Land-surface datum is 317.5 ft. above msl. Highest water level 64.89 below lsd, Aug. 31, 1949; lowest 72.78 below lsd, Oct. 26, 1961. Records available: 1949-61.

Fa:W-2. Formerly 24:10-2. Tennessee Division of Geology and U. S. Geol. Survey. Near Braden. Lat. 35°22′26″, long. 89°32′52″. Drilled artesian well in sand of Claiborne Group, diameter 6 to 4 in., depth 365 ft., screen 345-365. Land-surface datum is 317.2 ft. above msl. Highest water level 37.25 below lsd, Mar. 10, 1952; lowest 41.39 below lsd, Jan. 1, 10-11, 18-19, 1957. Records available: 1949-61.

Md:N-1. Formerly 57:36-1. Tennessee Division of Geology and U. S. Geol. Survey. Near Claybrook. Lat. 35°42′26″, long. 88°37′47″. Drilled artesian well in sand of Ripley Formation, diameter 6 to 4 in., depth 659 ft., screen 639-659. Land-surface datum is 562.7 ft. above msl. Highest water level 124.50 below lsd, Mar. 10, 1952; lowest 128.86 below lsd, Nov. 19, 1956. Records available: 1949-61.

Sh:O-170. Formerly 79:7-26. Memphis Light, Gas and Water Division. North Parkway and North Dunlap St. Lat. 35°09′11″, long. 90°01′49″. Drilled artesian well in sand of Wilcox Group, diameter 8 in., depth 1,387 ft. Land-surface datum is 255.4 ft. above msl. Highest water level 48.7 below lsd, Jan. 26, 1945; lowest 107.9 below lsd, Sept. 27, 1954. Records available: 1945-61.

Sh:P-76. Formerly 79:5-193. Memphis Light, Gas and Water Division. Central Ave. and Tanglewood St. Lat. 35°07′36″, long. 89°59′32″. Drilled artesian well in sand of Claiborne Group, diameter 12 in., depth 488 ft. Land-surface datum is 286.7 ft. above msl. Highest water level 58.65 below lsd, April 3, 1933; lowest 124.6 below 1st, Sept. 2, 1961. Records available: 1928-61.

Sh:Q-1. Formerly 79:1-3. Memphis Light, Gas and Water Division. Lat. 35°08′59″, long. 89°47′59″. Drilled artesian well in sand of Claiborne Group, diameter 6 in., depth 384 ft. Land-surface datum is 330.4 ft. above msl. Highest water level 74.08 below lsd, Dec. 27, 1940; lowest 87.51 below lsd, Oct. 27, 1961. Records available: 1940-61.

Wm:M-1. Formerly 94:1-1. Tennessee Division of Geology and U. S. Geol. Survey. Near Franklin. Lat. 35°55′02″, long. 86°54′11″. Drilled artesian well in Knox Dolomite, diameter 6 in., depth 1,160 ft. Land-surface datum is about 725 ft. above msl. Highest water level 84.21 below lsd, Mar. 10, 1952; lowest 114.81 below lsd, Jan. 31, 1950. Records available: 1950-61.

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THE NUMBER OF PLANES CONTAINED IN THE COMPLEMENT OF A QUADRIC IN AN AFFINE GALOIS SPACE

Eckford Cohen
Department of Mathematics, University of Tennessee

1. *Introduction.* Let q denote a power of an *odd* prime p,$q=p^n$, and let S_m denote the vector space of dimension m over the Galois field, F=GF(q). It will be shown elsewhere that in S_m, every quadric meets every hyperplane, provided $m>3$. By a quadric, we mean a central, nondegenerate quadric; that is, a surface generated by equating a quadratic form of S_m of determinant $\neq 0$ to an element of F.

The number of planes are determined which have no point in common with a given quadric of S_3 (see Theorem 2). The argument is purely algebraic in nature. The only tools required are some results on squares and sums of squares in a Galois field as used by Dickson (1901).

2. *Preliminaries.* Let P denote the prime field GF(p) and let Ψ denote the Legendre symbol in F; that is, for elements c in F,$\Psi(c)=1,-1$, or 0, according as c is a nonzero square, a non-square, or is the zero element of F.

Lemma 1 (cf. Dickson [1,§62]).[1] *If* c *is a non-square of* P, *then* c *is a square in* F *if and only if* F *is of even degree* n *over* P.

Lemma 2 (Dickson [1,§64]). *Let* α,α_1,α_2 *denote elements of* F, $\alpha_1\alpha_2\neq 0$, *and let* $B(\alpha)$ *denote the number of solutions in* F *of* $\alpha=\alpha_1 x_1^2+\alpha_2 x_2^2$. *Then* $B(\alpha)=q-\Psi(-\alpha_1\alpha_2)$ *or* $q+(q-1)\Psi(-\alpha_1\alpha_2)$ *according as* $\alpha\neq 0$ *or* $\alpha=0$.

In particular,

Corollary 1. $B(\alpha)>0$ for all α in F.

Lemma 3 (cf. Dickson [1,§168-169]). *Let* Δ *denote a given nonzero element of* F. *Every quadratic form* Q_m *of* S_m *with determinant* $\Delta\neq 0$ *can be reduced by a non-singular linear transformation of* S_m *to a form,* $\alpha_1 x_1^2+\dots+\alpha_m x_m^2$ $(\alpha_1\dots\alpha_m=\Delta)$; *moreover, in the case* m *odd,* Q_m *is congruent to the form,* $\Delta(x_1^2+\dots+x_m^2)$.

Remark 1. By Lemma 3, there is no loss of generality in assuming the given quadric of the paper to be of the form,

$$a=x_1^2+x_2^2+x_3^2, \tag{1}$$

where a is an arbitrary element of F.

Convention. Let $b,\beta_1,\beta_2,\beta_3$ denote elements of F where at least one β_i is different from 0. In order to ensure that the planes of our discussion,

$$b=\beta_1 x_1+\beta_2 x_2+\beta_3 x_3, \tag{2}$$

are distinct, it will always be assumed that the first nonzero β_i occurring in (2) has the value 1.

3. *The main results.* We shall need some additional notation and terminology. Let $N_1(a)$ denote the total number of planes (2) containing no point in common with the quadric surface (1). Let $N_2(a)$ denote the number of *homogeneous* planes (that is, planes (2) in which b=0) without a point in common with (1). Equivalently, $N_2(a)$ may be defined as the number of 2— dimensional subspaces of S_3 contained in the complements of (1). Let $N_3(a)$ denote the number of *oblique* planes (planes (2) with $\beta_1\beta_2\beta_3\neq 0$) which contain no point of the quadric (1). Finally, we divide the spaces S_3 into two classes, C and C′, where C consists of all S_3 for which either $p\equiv 1 \pmod 4$ or $p\equiv 3 \pmod 4$ and n is even, while C′ consists of those S_3 with $p\equiv 3 \pmod 4$ and n odd.

Theorem 1. *Any plane of* S_3 *contained in the complement of the quadric* (1) *is necessarily homogeneous. In fact,*

$$N_1(a)=N_2(a)=\begin{cases} q+1 \text{ if } \Psi(-a)=-1, \\ 0 \quad (\textit{otherwise}). \end{cases} \tag{3}$$

Moreover, in case $\Psi(-a)=-1$,

$$N_3(a)=\begin{cases} q-5 \text{ if } S_3\epsilon C, \\ q+1 \text{ if } S_3\epsilon C'. \end{cases} \tag{4}$$

Proof. We consider two cases, according as the plane (2) is or is not parallel to a coördinate axis.

Case 1 (Non-oblique planes). The plane

$$b=x_1+\beta x_2,\ \beta\epsilon F, \tag{5}$$

has a point in common with the quadric (1) if and only if the equation,

$$a-b^2=c x_2^2-2b\beta x_2+x_3^2, c=1+\beta^2, \tag{6}$$

has a solution (x_2,x_3) in F. The latter equation may be written

$$\frac{ac-b^2}{c^2}=\left(x_2-\frac{b\beta}{c}\right)^2+\left(\frac{1}{c}\right)x_3^2 \text{ if } c\neq 0, \tag{7}$$

and

$$a=b^2-2b\beta x_2+x_3^2 \text{ in case } c=0. \tag{8}$$

The equation in (7) is always solvable, by Corollary 1. It will be noted that the case c=0 implies that $\beta\neq 0$. Hence, if $b\neq 0$, (8) has a solution with any preassigned value for x_3. If b=0, (8) is insolvable if and only if a is a non-square of F.

Therefore, a necessary and sufficient condition that (5) have no point in common with (1) is that

$$b=0,\Psi(a)=-1,\Psi(-1)=1,\beta^2=-1, \tag{9}$$

in which case β has exactly two values, the two square roots in F of —1. Letting $N_1^*(a)$ denote the number of non-oblique planes (2) and $N_2^*(a)$ the number of non-oblique, homogeneous planes (2) having no point in common with the quadric (1), one obtains

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[1] Numbers in brackets refer to the bibliography.