A FORMULA FOR HEART LOAD[1]

NEUTON S. STERN, M.D.

DIVISION OF MEDICINE, UNIVERSITY OF TENNESSEE, SCHOOL OF MEDICINE

The heart is a pump. As such it has work to do, a load to carry, and insofar as it does work it must follow mechanical laws. In view of the variety of ways in which the work of the heart may be increased, it is of value to examine the factors underlying the load imposed upon the heart, and to determine if possible the effects of variations in these factors.

The prevailing opinion as evidenced in the literature is that the work of the heart may be computed simply from the output and the blood pressure. A recent example is the following quotation from Moore, Hamilton, and Kinsman[2]. "By effective work of the heart is meant roughly the mean blood pressure in grams per square centimeter times the output of the beat in cubic centimeter." Such statements fail to take into consideration all the factors involved.

Physicists state that the formula for the complete energy of a flowing fluid (neglecting viscosity and assuming stream line flow) is as follows:

$$\frac{mp}{d} + 1/2mv^2 + mgh.$$

The resultant of this formula is expressed in ergs or dyne centimeters. Let us consider this formula in the light of circulatory physiology. The various expressions will be explained in the course of the discussion.

Consider first mgh. This is the potential energy of the fluid due to its elevation h above some standard level. If the standard is the heart, this may be considered zero, since in the circulation all blood that goes above heart level eventually returns to it, and all blood that goes below eventually returns to it. All movement of blood away from the standard level is counter-balanced by an equal movement of blood in the opposite direction. To all intents and purposes therefore, the blood has no potential energy as determined by its height above the standard level, the heart, and hence $h = 0$. If $h = 0$, the expression mgh must be zero also, and may be entirely neglected in the consideration of our problem.

[1]Read before the Tennessee Academy of Science, Memphis, April 27, 1929.

[2]Moore, J. W., Hamilton, W. F., and Kinsman, J. M. The Ethyl Iodide Method for Determining the Circulation. Jour. Amer. Med. Assoc., 87:817, No. 11, September 11, 1926.

The first term $\frac{mp}{d}$ is the "pressure energy" and represents the work performed by lifting the fluid to a certain height against gravity. In the physiological problem, the blood pressure may be considered as the height to which the blood is lifted. In this expression d is the density of the fluid. It is understood, of course, that the specific gravity of blood is about 1.026. But the error in considering it as unity is small, and it will here be neglected. The term then reads mp.

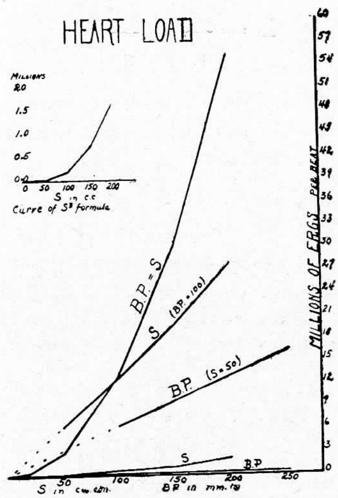

Fig. 1. Curves showing heart load under different conditions of blood pressure (B.P.) and stroke volume (S.).

M represents mass. Mass is determined by the ratio of volume to density, but if density equals 1, mass equals volume, or in our present problem, the number of cubic centimeters of blood put out by each beat of the heart. This is the stroke volume, nowadays a determinable factor. It will be represented by S in the final formula.

P is the pressure factor. In the physiological problem, it is partly represented by the blood pressure which is ordinarily expressed in millimeters of mercury. In order to change this reading into centimeters of water which is the unit necessary in this gram-centimeter formula, the factor 1.36 must be used.

Since the blood pressure is considered as a lifting force doing work against gravity, the factor g, the acceleration of gravity, 980, must be included in the formula to enable the final result to be expressed in dynes.

Therefore $$P = 980 \times 1.36 \text{ bp.}$$
$$= 1332.8 \text{ bp.}$$

Mp in the original formula may now be expressed as 1332.8 x bp x S, where 1332.8 is a constant, bp the blood pressure reading in millimeters of mercury, and S the stroke volume.

The remaining term in the original formula is the velocity component, $1/2\ mv^2$. Let us translate this into the physiological equivalents.

If the aorta were a rigid tube of uniform diameter throughout its length, the velocity could be calculated. If the blood mass put out by the heart per beat is divided by the area of the cross section of the aorta the result is the length of the column of blood per beat.

$$L = \frac{m}{a} = \frac{s}{a}$$

If this length is considered with reference to the time of the beat, the length per second may be found, i. e., the velocity.

$$V = \frac{L}{t} = \frac{s}{a} \times \frac{1}{t} = \frac{s}{at}$$

Where L = length of the column of blood in cm., s = output per beat in c. c. (stroke volume), a = cross sectional area of aortic orifice, in cm^2., v = velocity in cm. per second, and t = time in seconds (contraction time).

If $$V = \frac{s}{at},\ V^2 = \frac{s^2}{a^2t^2}$$

By substituting this value in the term $1/2\ mv^2$, and substituting s for m, the expression $1/2 \times s \times \frac{s^2}{a^2t^2}$ or $\frac{s^3}{2a^2t^2}$ is derived.

Instead of the formula $\frac{mp}{d} \times 1/2\ mv^2 \times mgh$ we may therefore write, using the physiological equivalents,

$$(1332.8 \times bp \times s) + \frac{s^3}{2a^2t^2} = \text{work per beat.}$$

There are in this expression four factors, each of which is determinable, or could be made so: s = stroke volume (output per beat of the heart), bp = blood pressure in mm. Hg., a = the cross section of the aorta in cm^2, and t = the systolic contraction time.

This formula represents not the total work of the heart, because in the total work would have to be included the other ventricle, the energy required in doing internal work, i. e., metabolic changes, and energy expended during the isometric period of contraction. What this formula does represent therefore is the external work accomplished by a single ventricle. To get the load carried by the right ventricle the formula must be reapplied.

If the work per minute is desired the formula must be multiplied by the rate of the heart, r, and divided by 60.

$$\frac{r}{60}\,(1332.8 \times by \times s) + \frac{s^3}{2a^2t^2} = \text{work accomplished per minute.}$$

It is of interest likewise to determine the power of the heart. Power is the rate of working and depends upon the time within which the work is accomplished. The longer the time used to accomplish a piece of work, the less power is necessary. The power is determined by dividing the work accomplished by the time in which it is done.

$$\frac{(1332.8 \times bp \times s)}{t} + \frac{s^3}{2a^2t^3} = \text{power (rate of work).}$$

With these formulae determined, let us substitute actual values to see what results may be obtained in figures. bp of course, is the blood pressure determined in the usual way by sphygmomanometer. Physiological values under various conditions of rest and effort will vary between 100 and 250 millimeters of mercury. S is the stroke volume of the heart. New methods have been devised for determining this and Burwell and Robinson[3] have found that an average stroke volume under basal conditions is 60 cc. According to Henderson[4], 30,000 cc. of blood may flow in a minute during vigorous exercise. At a pulse rate of 150, the stroke volume must be 200 cc. S may vary therefore in round numbers between 50 and 200 cc.

A is the cross sectional area of the aortic orifice. The average circumference of the aortic orifice in men is 8 cm[5]. The area of the cross section at this level then is 5.1 sq. cm. (approximately). As far as I know, no figures as yet correlate the internal measurements of the aorta, with the external ones which may be determined by X-ray.

T is the time in seconds of the duration of systolic contraction. Recent work of Lombard and Cope[6] shows that this time is variable, shortening (but not proportionally) as the pulse rate increases. They find that systole lasts from 0.3 to 0.2 seconds in men under varying conditions; 0.3 second may be taken as an average basal figure.

If these figures are substituted in the formula interesting curves are obtained (Fig. 1). In these curves, t is taken as 0.3 and a as 5.1 sq. cm.

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[3]Burwell, C. S., and Robinson, G. C. The Gaseous Content of the Blood, and Output of the Heart in Normal Resting Adults. Jour. Clin. Invest. 1:87, No. 1, October, 1924.

[4]Henderson, Y. Efficiency of the Heart and Its Measurement. Lancet, 2:1265, December 19, 1925.

[5]Mallory and Wright, Pathological Technique, 7th Edition, 1921, p. 491. W. B. Saunders and Company, Philadelphia.

[6]Lombard, W. P., and Cope, O. M. The Duration of the Systole of the Left Ventricle of Man. Amer. Jour. Physiol. 77:263, No. 2, July, 1926.

In the lowest curve marked BP., S = 1, and the B.P. increases from 1 to 250. In the curve just above it, B.P. is 1 and S increases from 1 to 200. These curves are really theoretical conditions as circulation does not continue with such low B.P. and S.

If we construct curves within physiological limits, we may use a stroke volume of 50 c. c. and blood pressure of 100 mm. Hg. as basal figures. The next curve shows the effect of increases in blood pressure with stroke volume constant. This is practically a straight line within these limits. The curve above shows the effect of increase in stroke volume with blood pressure constant. This line is gently curving upward, and shows a much more rapid increase in work than with increase in blood pressure. The top curve shows the extreme rapidity with which load increases when both blood pressure and stroke volume mount together.

If blood pressure is 100 mm. Hg., and the stroke volume 100 c. c. the work is 13.5 million ergs. If this work is accomplished in one contraction that lasts 0.3 second, the rate of working, or power of the heart is 45 million ergs per second, or 4.5 watts.

SUMMARY

The work formula for a pump has been expressed in physiological terms so as to calculate in ergs the load of work performed by one ventricle during each heart beat. The formula is

$$1332.8 \times bp \times s + \frac{s^3}{2a^2t^2}$$

The load per minute may be calculated by multiplying this term by $\frac{r}{60}$.

The power of the heart during systole may be determined by dividing the formula by t, systolic contraction time.

A study of substituted values in this formula suggests that:

Work increases in a straight line with increases in blood pressure at any given stroke volume, but not in proportion to the increase in pressure.

Work increases in a gradually rising curve with increases in stroke volume at any given blood pressure. Within physiological limits the curve is almost a straight line, but more rapidly rising than with increases in blood pressure.

Work increases with great rapidity when both blood pressure and stroke increase to their physiological limits.

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