TWENTY-FOURTH ANNUAL TENNESSEE JUNIOR ACADEMY OF SCIENCE

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The junior scientists in our high schools are given the opportunity to develop in a professional and scientific environment. This environment is provided primarily through the cooperation of our school teachers and the Tennessee Academy of Science. A student's early exposure and association with the Tennessee Junior Academy of Science (sponsored by TAS) program usually results in his continued interest in science.

It is through the TAS science programs for youth that the talented science student reveals himself. Through proper guidance from science and mathematics teachers and early association with professional scientists and their work, he is encouraged to explore the wonders of science. Furthermore, there is a matter which is extremely important: he receives recognition for a job well done. All the ingredients so necessary to his development somehow come together to produce the transformation from an inquisitive youth to a junior scientist. Although it is impossible to define all these ingredients, statistics indicate quite strongly their presence in the Tennessee Junior Academy of Science Program in sufficient abundance and order. The results of this year's programs offer further clear evidence of its continued effectiveness through the years.

The annual climax of the TJAS program is, of course, State Science Day which, this year, was held in Memphis. All efforts of students and teachers in science clubs, science seminars and lectures, science projects and papers, and Regional Science Day Programs all over the State come to a focus on State Science Day. Participation in this day's program represents a significant accomplishment for all selected students. In Tennessee, this program is one of the great "wonders" of our scientific community. For the teachers, scientists, engineers, and others who took part in the TJAS programs, there remains a sense of deep satisfaction and pride in the accomplishments of our junior scientists.

Five winners in the Senior Division of the Twenty-Fourth Annual TJAS were selected by the judges from the presentations of papers at Memphis. The winners presented excellent papers on their projects. It is hoped that means will be made available sometime for publishing the outstanding work accomplished by some of our science students.

Name and Address	Sponsor, School, City	Project Title
Bacon, Raymond W. 1613 Autry Way Knoxville, Tennessee	Harry H. Kroll Bearden High School Knoxville, Tennessee	The Role of Vision in the Maintenance of Balance in the Pigeon
Ergen, John A. K. 103 Orkney Road Oak Ridge, Tennessee	E. Powell Pickett Oak Ridge High School Oak Ridge, Tennessee	Inertia of the Eye
Eubanks, Sylvia 2001 Vine Street Chattanooga, Tennessee	Mrs. J. S. Woods Howard High School Chattanooga, Tennessee	A Finite Algebraic System
Goldstein, Elaine 1041 N. Belvedere Memphis, Tennessee	William T. Conn, Jr. Central High School Memphis, Tennessee	Use of Thin-Layer Chromatography
Hoover, Mark H. 1217 Garden Drive Kingsport, Tennessee	James T. Davis Dobyns-Bennett High School Kingsport, Tennessee	Extraction of Aluminum from Silicate Minerals

At least one of the winning papers deserves special recognition at this time for its excellence and thoroughness. Miss Sylvia Eubanks' paper "A Finite Algebraic System" is a tribute to her curiosity, ability and inventiveness in this area of mathematics. Congratulations, also, to Mrs. J. S. Woods, Miss Eubanks' teacher-sponsor, for her understanding and dedication to the development of her students.

Miss Eubanks' paper presents a new algebra based on modulo arithmetic using the integers modulo seven. In this algebra, the integers of the real-number system are separated into seven infinite sets of integers. These sets are identified by the remainders obtained after division of the real integers by the number 7. These sets are defined below:

$$\bar{0} = (.\,,\,.\,,\,-14, -7, 0, 7, 14, 21, .\,,\,.\,,\,.\,,)$$
$$\bar{1} = (.\,,\,.\,,\,-15, -8, 1, 8, 15, 22, .\,,\,.\,,\,.\,,)$$
$$\bar{2} = (.\,,\,.\,,\,-16, -9, 2, 9, 16, 23, .\,,\,.\,,\,.\,,)$$
$$\bar{3} = (.\,,\,.\,,\,-17, -10, 3, 10, 17, 24, .\,,\,.\,,\,.\,,)$$
$$\bar{4} = (.\,,\,.\,,\,-18, -11, 4, 11, 18, 25, .\,,\,.\,,\,.\,,)$$
$$\bar{5} = (.\,,\,.\,,\,-19, -12, 5, 12, 19, 26, .\,,\,.\,,\,.\,,)$$
$$\bar{6} = (.\,,\,.\,,\,-20, -13, 6, 13, 20, 27, .\,,\,.\,,\,.\,,)$$

By using these sets and defining the addition and multiplication of sets as shown below, the field properties of this system are demonstrated.

$\bar{x} + \bar{y}$

$\bar{y}$ \ $\bar{x}$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

$\bar{x} \cdot \bar{y}$

$\bar{y}$ \ $\bar{x}$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Subtraction and division are defined in terms of addition and multiplication, for example:

$\bar{0} - \bar{6} = \bar{1}$ because $\bar{1} + \bar{6} = \bar{0}$

$\bar{1} \div \bar{6} = \bar{6}$ because $\bar{6} \cdot \bar{6} = \bar{1}$

Next first degree algebraic equations in one and two variables are explored to use as building blocks for extension into the study of polynominal equations of degree up to and including 6. In these equations the coefficients represent elements in modulo 7. Solutions of these equations are presented. Equations of higher degree than 6 are shown to have graphs identical with those of some polynominal of degree less than 3.

Ingenious techniques are used in the solution of irrational equations. It is shown that not all quadratic equations have solutions in the field 0, 1, 2, 3, 4, 5, 6. For example the quadratic $x^2 + 1 = 0$ has no roots in the integers modulo 7. It was felt that this system did not have enough elements and that all equations should have a solution. Therefore $x^2 + 1 = 0$ was given a solution of "a." Hence

$x^2 + 1 = 0$
let $x = a$
hence $a^2 + 1 = 0$
$a^2 = 6$
$a = \sqrt{6}$

A new element is added to the modulo 7 integers: 0, 1, 2, 3, 4, 5, 6, a. In order to maintain the field property of closure 2a, 3a, 4a, . . . , a + 1, a + 2, a + 3, ... , 2a + 1, 3a + 1, 4a + 1 . . . . . , 2a + 2, 3a + 2, 4a + 2, ... , 2a + 3, 3a + 3, 4a + 3, . . . , 4a + 4, 4a + 5, 4a + 6, ... , 5a + 2, 5a + 3, 5a + 4, ... , 6a + 1, 6a + 2, 6a + 3, ... 6a + 6, must be in the set. Other elements a^2, a^3, a^4, a^5, and a^6 must also be in the set. However, $a^2 = 6$, $a^3 = 6a$, $a^4 = 1$, $a^5 = a$, $a^6 = 6$ are numbers already listed in the set. The enlarged set now has 49 elements.

This new set is also a field because there is closure for addition and multiplication. The commutative and associative laws hold for both operations and each element has an additive and multiplicative inverse.

All quadratic equations which previously did not have solutions now have solutions. The following procedure illustrates the procedure for solving these equations:

$x^2 + x + 4 = 0$
$x^2 + x + 2 + 2 = 0$
$x^2 + x + 2 = 5$
$(x + 4)^2 = 5$
$(x + 4) = \sqrt{5}$
$x + 4 = 3a$
$x + 4 = 4a$

for $x + 4 = 4a$,
$x = 4a + 3$
for $x + 4 = 3a$,
$x = 3a + 3$

In like manner it can be shown that all quadratic equations now have solutions.

A similar procedure was developed for the solution of irrational cubic equations. The set was enlarged by adding an element "b" which is the cube root of 2.

Some of the general conclusions of this study are: (1) this type of algebraic system can be created for any modulo system p where p is a prime, (2) there are no fractions or negative numbers in this system, and (3) the number of polynominals of nth degree, for $n > 1$, is given by $N = 6(7)^n$.