TABLE 4: *Suggested use of selected soil mapping units in the THR soil association.*

Soil Mapping Unit	Cropping System Which Would Be Used Continually If Only Net Profit Were Considered	Cropping Systems Used If Erosion Is Not Allowed to Exceed Soil Loss Tolerance	Percentage In: Continuous Row Crop	Cropping Rotation With Meadow: 4-Year	6-Year	Orchard-Grass	Forest	Roads and Railroads	Gullies	Permanent Stream	Total Percent
Ae	corn	corn	50.38	--	--	6.89	37.30	6.87	--	--	101.44
Ag	w-s	w-s, w-s, mmmm	--	--	--	61.86	24.74	6.44	--	3.61	96.65
Bg	og	og	--	--	--	28.76	52.69	--	22.04	--	103.49
Bk	corn	og	--	--	--	41.20	39.80	5.00	14.00	--	100.00
Bm	og	og	--	--	--	--	49.68	13.15	33.33	--	96.16
Bn	og	og	--	--	--	11.50	28.83	--	60.97	--	101.30
Bo	og	og	--	--	--	--	46.97	--	53.03	--	100.00
Ce	og	og	--	--	--	70.20	22.05	2.67	--	--	94.92
Eh	corn	corn	--	--	--	26.14	36.68	3.06	28.16	--	94.04
Ha	corn	corn	28.65	--	--	57.41	6.08	3.70	.88	1.33	98.05
He	corn	corn	--	--	--	100.00	--	--	--	--	100.00
Ho	corn	corn	100.00	--	--	--	--	--	--	--	100.00
Hr	corn	corn	--	--	--	13.28	31.12	3.69	26.55	19.63	94.27
Le	corn	corn	--	--	--	76.67	--	--	--	22.50	99.17
Ld	soybeans	soybeans	--	--	--	48.03	28.19	--	22.79	--	99.01
Nb	w-s	w-s	--	69.75	--	18.01	--	10.08	--	--	97.85
Te	corn	og	--	--	--	25.37	63.[illegible]	7.83	--	--	97.00
Tb	og	og	--	--	--	72.38	26.35	2.16	3.87	--	104.79

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A NOTE ON THE TORSION CONCEPTS OF LEVY AND GOLDIE

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ABSTRACT

In this note the rings R over which the torsion concepts of Levy and Goldie coincide are characterized. The Procesi-Small proof of Goldie's Theorem is used to show that these rings are precisely the semiprime (left) Goldie rings. The crux of our argument lies in showing that semiprime Goldie rings are characterized by the well-known properties (1) R has a left quotient ring, and (2) each essential left ideal of R contains a regular element.

INTRODUCTION

In Levy's torsion theory (Levy, 1963) an element m of a left R-module M is called a torsion element if $dm = 0$ for some regular element d of R. The set of torsion elements of each left R-module forms a submodule if and only if R has a left quotient ring. Goldie (1964) calls an element $m \in M$ a singular element if $Em = 0$ for some essential left ideal E of R; the set Z(M) of singular elements of M forms a submodule called the singular submodule of M. An element $m \in M$ is called a torsion element if there is an essential left ideal E of R with Em is a subset of Z(M); the set of torsion elements forms a submodule $Z_2(M)$ called the torsion submodule of M. We will use T(M) to denote the set of torsion elements of M under Levy's definition. These two torsion theories coincide if $T(M) = Z_2(M)$ for each left R-module M. We note that if this is the case, then T(M) is always a submodule, so R has a left quotient ring.

Goldie's theorem and a lemma.

The Procesi-Small paper (Procesi & Small, 1965) on Goldie's theorem contains the following results for a semiprime Goldie ring R. With the exception of (1), these are found in Goldie's paper (Goldie, 1960).

(1) R satisfies the descending chain condition on left annihilators.
(2) Each essential left ideal of R contains a regular element.
(3) R has a left quotient ring Q.
(4) Q is a semisimple left Artinian ring.

We also need the converse of Goldie's theorem which is found in (Goldie, 1960) or in Herstein's exposition (Herstein, 1968, p. 177) of the Procesi-Small paper:

(5) Let R be a left order in a semisimple left Artinian ring Q. Then R is a semiprime Goldie ring.

The following lemma shows that semiprime Goldie rings are chararterized by (2) and (3) above.

LEMMA 1. Let R be a ring with a left quotient ring Q. If each essential left ideal of R contains a regular element, then R is a semiprime Goldie ring.

By (5) above it suffices to show that Q is semisimple left Artinian. Interestingly the Procesi-Small proof of (4) above (Theorem 2 in (Procesi & Small, 1965)) serves this purpose since the only fact about R used in that proof is that each essential left ideal contains a regular element. The first paragraph of their proof establishes that Q is semiprime Goldie; the result (1) above is used to show Q is left Artinian.

Comparison of the torsion concepts.

We will use the following definition.

Definition 1. A submodule E of a left R-module M is said to be vital in M if for each $m \in M$ there is a regular element $r \in R$ such that $rm \in M$.

A left ideal A of a ring R is called vital if A is vital considered as a submodule of $_RR$. We will say A is integral if A contains a regular element. The following connections exist between essential, vital, and integral left ideals of R. Recall that a ring R has a left quotient ring iff R contains regular property (CM) Given a,r in (left) common multiple property (CM) Given a,r in R with r regular, there exist a', r' in R with r' regular, such that $a'r = r'a$.

(a) If A is vital, then A is essential.
(b) If A is vital, then A is integral.
(c) R satisfies CM ⇄ each integral left ideal of R is vital.
(d) R is semiprime Goldie → each essential left ideal of R is integral.
(e) R is semiprime Goldie → each essential left ideal of R is vital.

The proofs of (a), (b), and (c) are immediate, while (d) is a restatement of (2) Since a semiprime Goldie ring satisfies CM, (e) follows from (c) and (d). Using these facts and Lemma 1 it is easy to prove that R is semiprime Goldie if and only if the essential, vital, and integral left ideals of R coincide.

Definition 2. If E is a submodule of a left R-module M, we define $V(E) = \{m \in M \mid rm \in E$ for some regular $r \in R\}$.

Note that E is vital in M iff $V(E) = M$. If R has a left quotient ring it is easy to show that V(E) is a submodule (using CM). In this case V(E) is the largest submodule of M in which E is vital.

LEMMA 2. Let R be a ring with a left quotient ring and suppose $Z_2(M)$ is a subset of T(M) for each left R-module M. Then each essential left ideal E of R is vital (hence integral)

Proof. Suppose E is not vital. Then V(E) is a proper submodule of $_RR$. It is easy to see that M = R/V(E) is torsion-free under Levy's definition, i.e. T(M) = (0). But consider a nonzero element x + V(E) of M. The annihilator of this element is essential since it contains the essential left ideal $Ex^{-1} = \{r \in R \mid rx \in E\}$ of R. Thus $0 \neq x + V(E) \in Z_2(M)$, and we have a contradiction of the hypothesis $Z_2(M)$ is a subset of T(M).

THEOREM. The following conditions concerning the ring R are equivalent:

(1) R is semiprime Goldie.
(2) R has a left quotient ring, and each essential left ideal of R is integral.
(3) $T(M) = Z_2(M)$ for each left R-module M.

Proof. The equivalence of (1) and (2) was shown above. We will prove (2) $\rightleftarrows$ (3). Assume (2) and let M be a left R-module. Using the fact that each essential left ideal of R contains a regular element it is easy to see that $Z_2(M)$ is a subset of T(M). Let $m \in T(M)$ and let r be a regular element of R such that rm = 0. The left ideal Rr of R is essential due to the common multiple property of R, and Rr annihilates m; therefore $m \in Z_2(M)$. Thus (2) $\rightleftarrows$ (3).

Now assume (3). As noted before, this implies R has a left quotient ring. Lemma 2 shows that each essential left ideal of R is integral.

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APC Board Meets In Oak Ridge

The Air Pollution Control Board adopted a proposed revision of regulations for sulfur dioxide emissions at a regular meeting of the Board held in Oak Ridge on October 23. The revision provides for new county classifications, each county in Tennessee being classified into one of six classes. Each class has been established with the limit necessary to attain or maintain ambient air quality standards.

In other action by the Board, a request for a revision of a Board Order for Holston Army Ammunition Plant, Kingsport, was granted; consideration of a request from Beaunit Corporation, Elizabethton, for a variance was postponed until the next meeting; an extension of the Board Order for Tennessee Forging Steel, Harriman, was approved; a compliance schedule was approved for Lodge Manufacturing Company, South Pittsburg; an extension of an Order and a renewal of a variance were granted to Tennessee Eastman Company, Kingsport; and a variance for Aluminum Corporation of America, Alcoa, was granted.

The Board also heard a statement from TVA officials on the status of fly ash control at the Bull Run Steam Plant and statements from residents of the Claxton community near the Bull Run Plant on effects of fly ash fallout in the area.