found in New York, Saltville, Virginia, and Charleston, West Virginia, and salt was successfully prepared from sea water on Cape Cod from 1776. Courses in chemistry were offered at Pennsylvania, King's (Columbia), and William and Mary before 1776.

Lavoisier's recognition of the role of air in combustion, replacing the phlogiston theory; his extensive quantative experiments; and his Traité de Chimie (1789) were the beginning of modern chemistry. Earlier texts had a completely different vocabulary and conceptual basis, but his text has been described as sounding like an old-fashioned modern text. Lavoisier's synthesis, in spite of its fundamental advance, had many precursors and contributors, as in all scientific revolutions. Van Helmont (1579-1644), Robert Boyle (1627-1691, of Boyle's law), John Mayow (1641-1679), Stephen Hales (1677-1761), the "pneumatic chemists," Joseph Black (1725-1799), Joseph Priestley (1733-1804, who discovered oxygen—a couple of years after the Swedish chemist Scheele—but who remained a phlogistonist), and the Russian Mikhail Lomonosov (1711-1765, who anticipated the idea of combustion by 1750), and many others contributed to the change from alchemy to chemistry. Priestley emigrated to America in 1794, but the "new" chemistry reached America mainly from France and particularly from Scotland, where Lavoisier's ideas were promptly accepted.

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MATHEMATICS IN 1776*

ALVIN TIRMAN
East Tennessee State University
Kingsport, Tennessee 37660

ABSTRACT

An overview is presented of the history of mathematics as it developed in the eighteenth century in Europe and Colonial America.

When the eighteenth century opened, England's Isaac Newton (1642-1727)—perhaps the greatest scientist ever to draw breath on our planet—was alive and bitter. It seemed to him that Gottfried Leibniz (1647-1716) was a sneak, a cheat, and a contemptible plagiarist. He was convinced that the German had peeked at manuscripts of his unpublished mathematics, and he believed that Leibniz had taken the results and had printed them as his own. To Leibniz, these accusations seemed unwarranted and unfair. Although Leibniz probably had seen portions of Newton's work on fluxions and fluents, his own formulations of the calculus arose out of Cavalieri's (1598-1647) geometrical approach and differed so strongly from Newton's mechanical one that only a bigotted blockhead could have accused him of stealing another man's thoughts.

In this typical scientific dispute—a dispute in which the learned disciples of both men misunderstood the match but not the personalities—the result of the conflict was predictable. The English jingoists rallied behind Newton; the continental xenophobes sided with Leibniz. The denser their dumbness, the surer and louder their loyalties. Consequently, the dispute prospered. Although it ill served the British, it aided the Europeans and shaped the history of mathematics in the eighteenth century. Newton's x (dot) fluxions of x fluents and x (double dot) fluxions of x (dot) fluents was brilliant in conception but brainless in pedagogy. The superiority of the notation devised by Leibniz soon became evident. Using dx, dy, dy/dx and ∫dx, European students progressed readily and rapidly through the calculus and gave us a golden age of mathematics. The English hung with Newtonian notation and died mathematically.

To Newton, the calculus was a tool. Only out of a necessity to solve certain problems in mechanics had he devised it. He fathered the calculus and the study of differential equations because they seemed to describe natural phenomena, particularly motion. However, the far-reaching results of his work extended well beyond the confines of simple mechanics. In addition to rate of change studies, the calculus soon proved indispensible as an engineering aid in finding areas of surfaces, volumes of solids, centers of gravity, moments of inertia and strengths of materials. It was also employed in statistics; geometrically, it could be used to study curves, to find their maxima and minima and their points of inflection.

The foundations of the calculus rested on shaky ground which would not become secure until a later century, when genuises such as Cauchy, Gauss, Abel and Bolzano would pioneer insistence on rigor in mathematics. In 1734, Bishop George Berkeley (1685-1753), one-time resident of Rhode Island, assailed the assumptions and the abstruseness of the calculus. In his famous critique, *The Analyst,* he railed against the new mathematics as being "beyond the evidence of our senses and our understanding;" he accused his scientific opponents, just as they had charged the adherents of religion, of "submitting to authority, taking things on trust, and believing points inconceivable." Unable to contradict or confound Berkeley, his antagonists ignored him.

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Mathematics, after all, had no need for a spokesman: Mathematics built bridges.

In the eighteenth century, Europe hatched a flock of fine mathematicians. Foremost were Jakob Bernoulli (1654-1705), Johann Bernoulli (1667-1748), Leonard Euler (1707-1783), Joseph Lagrange (1736-1813) and Pierre Laplace (1749-1827). Their contributions furthered understanding of the calculus and extended the range and usefulness of applied mathematics. Despite these accomplishments, their fame suffered a fate not unlike that of the conquistadors who followed Columbus. Their efforts were dwarfed in grandeur by those of the giant who preceded them. Neither Descartes nor Pascal nor Fermat possessed or displayed more mathematical talent than Lagrange or Euler. Nonetheless, these seventeenth-century scholars traditionally have received a favorable press. They had great wisdom. They were born before Newton.

The brothers Jakob and Johann Bernoulli were reared in the city of Basel in Switzerland. Possessing a common background, they shared a common rivalry. Each studied under Leibniz and each became a fountain of new mathematics. Among the contributions of Jakob were the use of polar coordinates, the study of the catenary, the lemniscate and the logarithmic spiral. He became honored eponymously for his findings: the "Bernoulli Equation," the "Bernoulli Numbers," and the "Theorem of Bernoulli on Binomial Distributions" are monuments to him.

As for Johann, he showed wizardry in solving differential equations. In recognition of his work on the brachystochrone, the curve of quickest descent in a gravitational field, he has been classed as the creator of the calculus of variations. The brachystochrone problem was solved with the cycloid, the curve that also solved the tautochrone problem—the path along which a particle in a gravitational field reaches the lowest point in a time independent of its starting point.

The Bernoulli brothers were not the only geniuses in the family. Johann's son, Daniel, was also an accomplished and creative mathematician, especially in partial differential equations. However, his finest work was in hydrostatics. He gave us the Bernoulli Principle and also formulated the kinetic theory of gases.

Through three generations the Bernoullis produced eight mathematicians of eminence; afterwards, in other fields, the family continued to grace the earth with great men.

The most prolific, versatile, and inventive mathematician of the age was Leonard Euler (1707-1783). Surviving without psychological damage his childhood in a small Swiss village as a Calvinist minister's son, Leonard learned from his father, in addition to the catechism, the calculus. Pastor Euler had studied the subject under Jakob Bernoulli at Basel. Consequently, when Leonard matriculated in theology at his dad's alma mater, his proficiency in math attracted the attention of Jakob's successor, Johann Bernoulli. The latter consented to give young Euler private help sessions once each week. They proved profitable. By the time Leonard had secured his master's degree, the Bernoulli clan had convinced Pastor Euler that his brilliant seventeen-year-old son had a calling—but in mathematics, rather than in the ministry.

Euler journeyed to St. Petersburg to join the Bernoullis, Daniel and Nicolaus, who had obtained a post for him in the recently formed Imperial Academy of Science. Except for his stay with Frederick the Great at the Berlin Academy from 1741 to 1766, Euler remained at the Imperial Academy throughout his life. He was amply rewarded. In Europe, during that age, scholars, scientists, poets and philosophers were regarded as national assets and were respected and remunerated accordingly. Euler, when he returned to Russia, received from Catherine the Great a fully furnished house, her personal cook and sufficient funds to care generously for himself and his eighteen dependents. Euler was adept at all kinds of multiplication.

In every area of mathematics which existed in the eighteenth century, Euler made signal contributions. His productivity never has been rivaled. Possessed with a near-photographic memory and endowed with a nimble mind, he poured forth an avalanche of articles and texts despite the loss of one eye in 1738 and blindness in the other in 1760. Nothing in a mathematician's career could have seemed more tragic. But Euler took it in stride. Of the 560 books and publications he authored, half were written after his vision failed (Struik, p. 168).

Leonard Euler became the most prolific number theorist of all time. In applied fields, his output was equally remarkable. He advanced the mathematical methods for the direct solution of problems in mechanics, astronomy, navigation, geography, geodesy, hydraulics, ballistics, insurance, and demography. To pure mathematics, he gave twenty-nine volumes of his total work; seventeen of these were in analysis. He was a founding father of the calculus of variations, the theory of differential equations, the theory of functions of a complex variable and the theory of special functions. His algorithms and the simple notation he devised have remained current in our trade until today. Symbols such as $f(x)$, Δx, Σ, e, and ι were Euler's. The texts he wrote became the texts of his contemporaries, their children, their children's children, and their children's children's children. His feats endured. They were Homeric.

Euler's only rival for supremacy in eighteenth-century mathematics was Joseph Louis Lagrange (1736-1813). Appointed Professor of Mathematics at the Artillery School of his native city, Turin, Italy, at the age of nineteen, Lagrange, in the same year, sent to Euler in Berlin a new method for handling the calculus of variations. The kindly Swiss was delighted with the report and withheld from publication findings of his own to assure the young genius full credit for the discoveries. Afterwards, he prevailed upon his colleagues to elect Lagrange a foreign member of the Berlin Academy. When Euler returned to Russia, he recommended to Frederick the Great that the thirty-year-old Lagrange be named his successor. The ruler agreed. Lagrange, on his arrival at court, greeted Frederick as "the greatest King in Europe;" Frederick, in reply, welcomed him as "the greatest mathematician in Europe." This was premature: Euler still had seventeen years to live.

Lagrange stayed on in Berlin for twenty years. While there, he completed his masterpiece, a diagramless text on mechanics.

To experts in physics and in "pure mathematics," Lagrange's disdain for geometry set a standard for clarity. Among mere mortals, it may have inspired less praiseworthy estimates.

Pierre Laplace was the last of the great eighteenth-century mathematicians. His five volumes on Celestial Mechanics, begun in 1799 and concluded in 1825, completed the triumph of Newtonian physics. His work in probability also was outstanding. But to students of math and science his fame will endure forever. He was the coiner of that famous phrase "It is easily seen that. . . ."

As children of the Bicentennial, we ask today: What were the contributions made to math by our Revolutionary forefathers? The answer is simple: None. American scholars of the Colonial past often were, as are American scholars today, regarded by their countrymen with suspicion, scorn and contempt. Two centuries ago, opposition to the idea of public education was widespread on the grounds that "it made boys lazy, dissatisfied with farm life, and led to religious skepticism." In at least one respect, learning had regressed. In Massachusetts in the seventeenth century, the standard of education was that every town of one hundred families had a grammar school. In the eighteenth century, only a town with over two hundred families met this requirement (Smith and Ginsburg, p. 15).

For mathematics, this poverty in education was not without redemption. In America, as on the continent of Europe, real mathematics was beyond the interest of elementary schools and outside the concern of colleges. It was through learned societies, publications and private instruction that math spread.

The first American scholar of reasonable mathematical skill was Isaac Greenwood (1702-1745). He studied in England and returned to the Colonies to teach at Harvard, where he was Hollis Professor of Math from 1728 to 1738. He was aquainted with the calculus and taught elements of the Newtonian fluxion to his students. Greenwood had begun his education as a divinity student; after succeeding to the ministry he displayed, in addition to a talent for math, a taste for alcohol. The behavior of Greenwood the occasional drunk, spiritual forefather of Timothy Leary, upset the puritanical bastions of the Harvard Administration. Predictably, they fired him. Deprived of his livelihood, Greenwood died soon afterwards. As a legacy, he left his countrymen permanently in his debt; for he had authored and published, in 1729, an elementary math text, *Arithmetic, Vulgar & Decimal.* It was the first decent text of its kind in English in North America, but was used sparsely throughout the Colonies.

Despite the loss of Greenwood, Harvard maintained the lead in Colonial mathematics with the appointment of John Winthrop (1714-1779) as Hollis Professor of Math. Though Winthrop was well trained in astronomy, he was no brighter than Greenwood. The math courses he taught were seldom demanding.

In the Colonies, as in the early United States, mathematics at the collegiate level approximated but did not equal that offered the average eleventh-grade student in high school today. Ivy League entrance requirements demanded only a knowledge of the four basic operations of arithmetic: addition, subtraction, multiplication and division. These requirements may seem absurdly low, but they are higher than those needed at present for graduation from East Tennessee State.

A word about the arithmetic texts used in North America. Before Greenwood's book, they were pitiful; afterwards, they remained the same. The most popular text was James Hodder's *English Arithmetic,* revised through twenty-five editions by the year 1714. In 1719 a reprint of the text was made and it contained the interesting note that it had had "above a Thousand Faults Amended." So much for its reliability (Smith and Ginsburg, p. 37).

In the Colonies a few learned societies sprang up in imitation of those in Europe. Benjamin Franklin founded in 1743 the American Philosophical Society, intending it to do for America what the Royal Society was doing for the British Isles. It offered two publications: the *Transactions* (from 1771) and *Proceedings* (from 1838). Of equal prominence was the American Academy of Arts and Sciences, begun during the Revolutionary War in 1780. Understandably, it mimicked the French Academy. Though both societies demanded high standards for membership and publications, none could be maintained in math. The years leading to the Revolution were of dismal worth in American science and math, but those afterwards were worse. One hundred years ago, during our nation's Centennial, President F.A.P. Barnard of Columbia University summed up the situation saying sadly, "In any review of the progress of science, the period which lies between the Declaration of Independence and the close of the eighteenth century may, without danger of important omission, he passed over in silence." (Smith and Ginsburg, p. 17).

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