JOURNAL OF THE TENNESSEE ACADEMY OF SCIENCE

VOLUME 54, NUMBER 1, JANUARY, 1979

A SUFFICIENCY CONDITION FOR POINTWISE CONVERGENCE

LYNDELL M. KERLEY

East Tennessee State University
Johnson City, Tennessee 37601

ABSTRACT

A sufficiency condition for pointwise convergence of Walsh-Fourier series is established.

INTRODUCTION

Zygmund contains the following classical theorem for trigonometric Fourier series which was established by Lebesgue.

Suppose

(a) $\int_0^h |F_x(t)|\,dt = o(h)$ as $h \to 0$ where $F_x(t) = (f(x+t)+f(x-t)-2f(x))/2$

and

(b) $\int_{\pi n^{-1}}^{\pi} |F_x(t) - F_x(t+\pi n^{-1})|\, t^{-1}dt = o(1)$ as $n \to \infty$.

Then $S_n(f,x)$ converges to $f(x)$ at each $x \in [0,\pi]$ and if (b) holds uniformly on $[c,d] \subseteq [0,\pi]$, then $S_n(f,x)$ converges uniformly to $f(x)$ on $[c,d]$.

The author will establish a theorem which is the Walsh-Fourier series analogue of Lebesque's result.

RESULTS

For the sake of clarity, the following definitions relating to Walsh-Fourier series will first be stated.

Definition 1. Let n be a non-negative integer. The Radamacher function r_n is defined for each real number

$$r_n(x) = \begin{cases} 1 \text{ if } x \in [2p2^{-n-1}, (2p+1)2^{-n-1}) \text{ for some } p = 0,1,\dots,2^n-1, \\ -1 \text{ if } x \in [(2p+1)2^{-n-1}, 2(p+1)2^{-n-1}) \text{ for some } p = 0,1,\dots,2^n-1. \end{cases}$$

Definition 2. The Walsh Function W_0 is identically 1. For each positive integer n,

$$W_n(x) = r_{n_1}(x) r_{n_2}(x) \cdots r_{n_s}(x),\ x \in (-\infty,\infty)$$

where $n = 2^{n_1} + \cdots + 2^{n_s}$ and $n_1 > n_2 > \cdots > n_s \geq 0$.

Definition 3. $S_n(x) = S_n(f,x) = \sum_{k=0}^{n-1} c_k W_k(x)$ where $c_k = \int_0^1 f(t) W_k(t)\,dt$.

Definition 4. $D_n(x) = \sum_{k=0}^{n-1} W_k(x)$.

For a study of many of the properties of Walsh-Fourier series, we refer the reader to Fine. Interestingly enough, many of the results that have been established for Walsh-Fourier series have older trigonometric analogues. The proofs of their results, however, are not routine extensions of the trigonmetric. Instead, the proofs often take on a flavor peculiar to the study of Walsh series.

In the statement of the theorem which follows, there is a striking similarity between it and Lebesgue's result. This is even more readily seen since the following identity holds:

$$f(x\dot{+}t) - f(x) = (f(x\dot{+}t) + f(x\dot{+}(-t)) - 2f(x))/2.$$

(a) $\int_0^{2^{-n}} |F_x(t)|\,dt = o(2^{-n})$ as $n \to \infty$ where $F_x(t) = f(x\dot{+}t) - f(x)$

and

(b) $\int_{2^{-n}}^{1} |F_x(t) - F_x(t + 2^{-n-1})|\, t^{-1}dt = o(1)$ as $n \to \infty$.

Then $S_n(f,x)$ converges to $f(x)$ at each $x \in [0,1)$. Also, if f is continuous on $[c,d] \subseteq [0,1)$ and (b) holds uniformly on $[c,d]$, then $S_n(f,x)$ converges uniformly to $f(x)$ on $[c,d]$.

Proof. For every positive integer n, there is a k which satisfies $2^k \leq n < 2^{k+1}$. Put $m = n - 2^k$. Then $m \geq 0$ and $m < 2^k$. Fine has shown that $D_n = D_{2^k} + W_k D_m$. In particular,

$$S_n(f,x) - f(x) = \int_0^1 F_x(t) D_n(t)\,dt$$

$$= \int_0^1 F_x(t) D_{2^k}(t)\,dt + \int_0^1 F_x(t) W_{2^k}(t) D_m(t)\,dt$$

$$A_k + B_k.$$

$$A_k = 2^k \int_0^{2^{-k}} F_x(t)\,dt;$$

consequently,

$$|A_k| \leq 2^k \int_0^{2^{-k}} |F_x(t)|\,dt$$

$$= 2^k o(2^{-k})$$

$$= o(1) \text{ as } k \to \infty. \quad (1)$$

Moreover,

$$B_k = \int_0^1 F_x(t) W_{2^k}(t) D_m(t)\,dt$$

$$= \int_0^{2^{-k}} F_x(t) r_k(t) D_m(t)\,dt + \int_{2^{-k}}^1 F_x(t) W_k(t) D_m(t)\,dt$$

$$= B_k^{(1)} + B_k^{(2)}.$$

Before estimating B_k, we examine D_m in more detail. Now

$D_m(t) = \sum_{j=0}^{m-1} W_j(t)$, and for each integer j, subject to the restriction $0 \le j < m < 2^k$, the corresponding Walsh function, W_j, is constant on $[p2^{-k}, (p+1)2^{-k})$, for $p = 0,\ldots,2^m - 1$. Hence $t \in [2p2^{-k-1}, (2p+1)2^{-k-1})$ implies $D_m(t) = D_m(t+2^{-k-1}) = D_m(2p2^{-k-1})$.

Using this fact and a change of variables, we obtain

$$B_k^{(2)} = \int_{2^{-k}}^{1} F_x(t)W_k(t)D_m(t)dt$$

$$= \sum_{p=1}^{2^k-1} \left(\int_{2p2^{-k-1}}^{(2p+1)2^{-k-1}} F_x(t)D_m(t)dt - \int_{(2p+1)2^{-k-1}}^{(2p+2)2^{-k-1}} F_x(t)D_m(t)dt \right)$$

$$= \sum_{p=1}^{2^k-1} \left(\int_{2p2^{-k-1}}^{(2p+1)2^{-k-1}} F_x(t)D_m(t)dt - \int_{2p2^{-k-1}}^{(2p+1)2^{-k-1}} F_x(t+2^{-k-1})D_m(t)dt \right)$$

$$= \int_{2^{-k}}^{1} (F_x(t) - F_x(t + 2^{-k-1}))D_m(t)dt.$$

Furthermore, we know from Fine that $|D_m(t)| \le t^{-1}$ for $t \in (0,1)$. Hence by hypothesis, we conclude that

$$|B_k^{(2)}| \le \int_{2^{-k}}^{1} |F_x(t) - F_x(t + 2^{-k-1})| t^{-1}dt$$

$$= o(1) \text{ as } k \to \infty. \qquad (2)$$

Estimating $B_2^{(1)}$ is easier. Indeed, since $|D_m(t)| \le m$ and $m < 2^k$, we conclude that

$$|B_k^{(1)}| \le \int_0^{2^{-k}} |F_x(t)|\,|D_m(t)|\,dt \le \int_0^{2^{-k}} m|F_x(t)|\,dt < 2^k \int_0^{2^{-k}} |F_x(t)|\,dt = 2^k o(2^{-k}) = o(1) \text{ as } k \to \infty.$$

Therefore from (2) and (3), we see

$$B_k = o(1) \text{ as } k \to \infty.$$

Then from (1) and (4), it follows that

$$S_n(f,x) - f(x) = o(1) \text{ as } k \to \infty.$$

LITERATURE CITED

N. J. Fine, "On the Walsh Functions," *Transactions of the Amer. Math. Soc.* 67 (1949), 372-414.

R. A. Hunt, "Developments related to the a.e. convergence of Fourier series," *Studies in Harmonic Analysis of the Math. Assoc. of Amer. 13* (1976), 20-37.

A. Zygmund, *Trigonometric Series, Vol. I and II*, New York: Cambridge University Press, 1959.

JOURNAL OF THE TENNESSEE ACADEMY OF SCIENCE

VOLUME 54, NUMBER 1, JANUARY, 1979

SPECTROPHOTOMETRIC DETERMINATION OF NITRITE IN HUMAN SALIVA

CARL T. BAHNER

Walters State Community College
Morristown, Tennessee 37814
and
Carson-Newman College
Jefferson City, Tennessee 37760
and

TRUETT C. PATTERSON, TIMOTHY SHANDS, VALERIE ADAMS, ROBYN AGRI, SAMUEL HOVIS, AND PATTY WARNER
Carson-Newman College
Jefferson City, Tennessee 37760

ABSTRACT

Nitrites are known to form carcinogenic and/or mutagenic compounds under certain conditions. A modified diazotization and coupling method applied to diluted saliva samples has been found convenient for screening tests, and for identifying factors that produce marked changes in nitrite levels. Among students in two East Tennessee colleges, saliva nitrite levels were somewhat lower than for the Massachusetts group studied by Tannenbaum et al., but almost all did have significant amounts. Among factors affecting these levels, eating turnip and mustard greens as part of a regular meal increased the nitrite levels more than three-fold even as long as three hours after the meal.

INTRODUCTION

In 1974, Tannenbaum, Sinskey, Weisman and Bishop reported that a study of saliva samples from more than 100 individuals revealed an average of 6 mg of nitrite per liter of saliva, corresponding to four times the estimated intake of nitrite from exogenous sources. Their observation prompted the authors to begin a study of the nitrite concentrations in the different population group available to us. Damage to hemoglobin by nitrites has been known for years, but the possibility that nitrites react in the digestive tract with amines to form carcinogenic nitrosamines causes even greater concern (cf. Wishnook, 1977). Some nitrosamines are so potent that even traces might lead to a significant number of cases of cancer.

MATERIALS AND METHODS

A simple, easy method of analysis for the nitrite concentration in saliva was needed in order to make practical surveys of various groups of persons to determine whether there are significant differences between high risk and low risk groups and to detect causes of major changes in nitrite levels. In view of the range of biological variation anticipated, extreme precision was not required. Colorimetric analysis seemed most likely to provide the convenience needed. After comparison of several analytical methods (Sawicki, Stanley, Pfaff and D'Amico, 1963), a version of the Griess diazo reaction (cf. Jacobs and Hochheimer, 1958; Kamphake, Hannah, Cohen, 1967) was worked out.

The color reagent contained 0.3 g of 1-naphthylethylenediamine hydrochloride, 6.0 g of sulfanilamide and 40 ml of concentrated phosphoric acid per 100 ml of aqueous solution. Working reagent was prepared by mixing 4.6 ml of 1.0 N NaOH with 16 ml of color reagent. Saliva samples were collected in small plastic containers and diluted with nine or more times their weight of distilled or deionized water. A blank consisting of 3.5 ml of the diluted saliva and 1.0 ml of water was used to set the spectrophotometer zero and 1.0 ml of working reagent was added to 3.5 ml of diluted saliva for the reading at 520 nm. A small correction was applied for absorption by the working reagent. The absorbance obeyed Beer's Law satisfactorily over the range to be used.

Dilution factor: Since it was necessary at times to dilute by different factors, a sample of saliva was divided into four parts and each part was diluted by a different factor. The results shown in Table 1 indicate both the minor variation introduced by change in dilution and the uniformity of sampling.

TABLE 1: *Effect of dilution factor on quantity of nitrite found.*

Dilution Factor	$NaNO_2$ Found
10	3.2 ppm
15	3.2
20	3.3
25	3.7

Delay factor: It was expected that the measured nitrite level would undergo some change on prolonged standing. A sample of saliva was diluted by a factor of 10 and divided into 9 portions which were analyzed at ten minute intervals. Results show that the change was negligible during the first thirty minutes. The stability of the color was tested also and found not to present any problem up to at least sixty minutes.

Effect of centrifuging: Nineteen samples were collected and diluted in the usual way. Each sample was then divided into two portions and one portion was centrifuged fifteen minutes in a Vari-Hi Speed Centricone Centrifuge; then both samples were analyzed. Ninety percent of the centrifuged samples gave results in the range of 1.00 to 1.23 times those for uncentrifuged ones, with a median ratio of 1.09/1.00.

RESULTS AND DISCUSSION

A set of samples provided by twenty 16-18 year old high school students who were eating regularly in the Carson-Newman College cafeteria during the sumemr of 1975 were tested by the modified Griess method using samples centrifuged after 10 fold dilution with the results shown in Figure I. Note that 20% of the group had nitrite levels below 2 ppm and the highest 20% had levels about 14 ppm, while the median was 6.3 ppm.

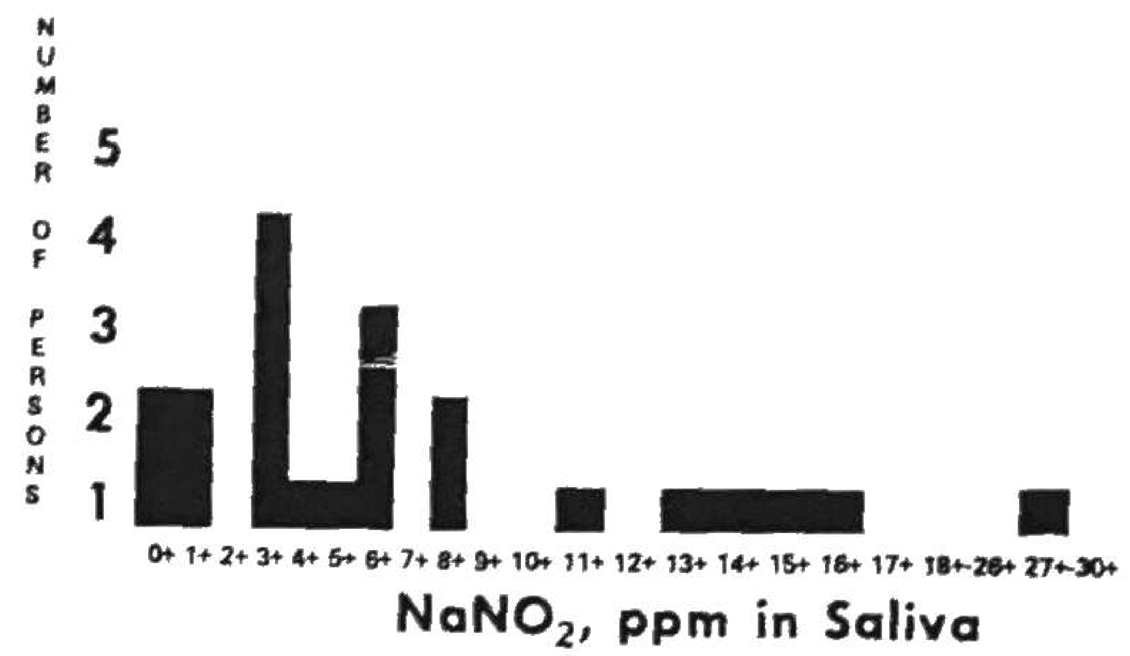

FIG. 1: *Nitrite content of saliva samples from 20 high school students who ate in the Carson-Newman Cafeteria.*

Text-figure 2 shows nitrite concentrations found in saliva samples from 78 individuals associated with Walters State Community College. Specimens were taken one hour or more after meals. Subdivision into groups did not reveal marked differences on the basis of time when samples were taken (morning or afternoon), sex, county of residence, or age. In view of the fact that carcinogens may be especially dangerous when administered to young animals, it is a matter for concern that the eight small children, ages 3-6, had nitrite levels of 0-19 ppm in saliva samples collected two hours after breakfast.

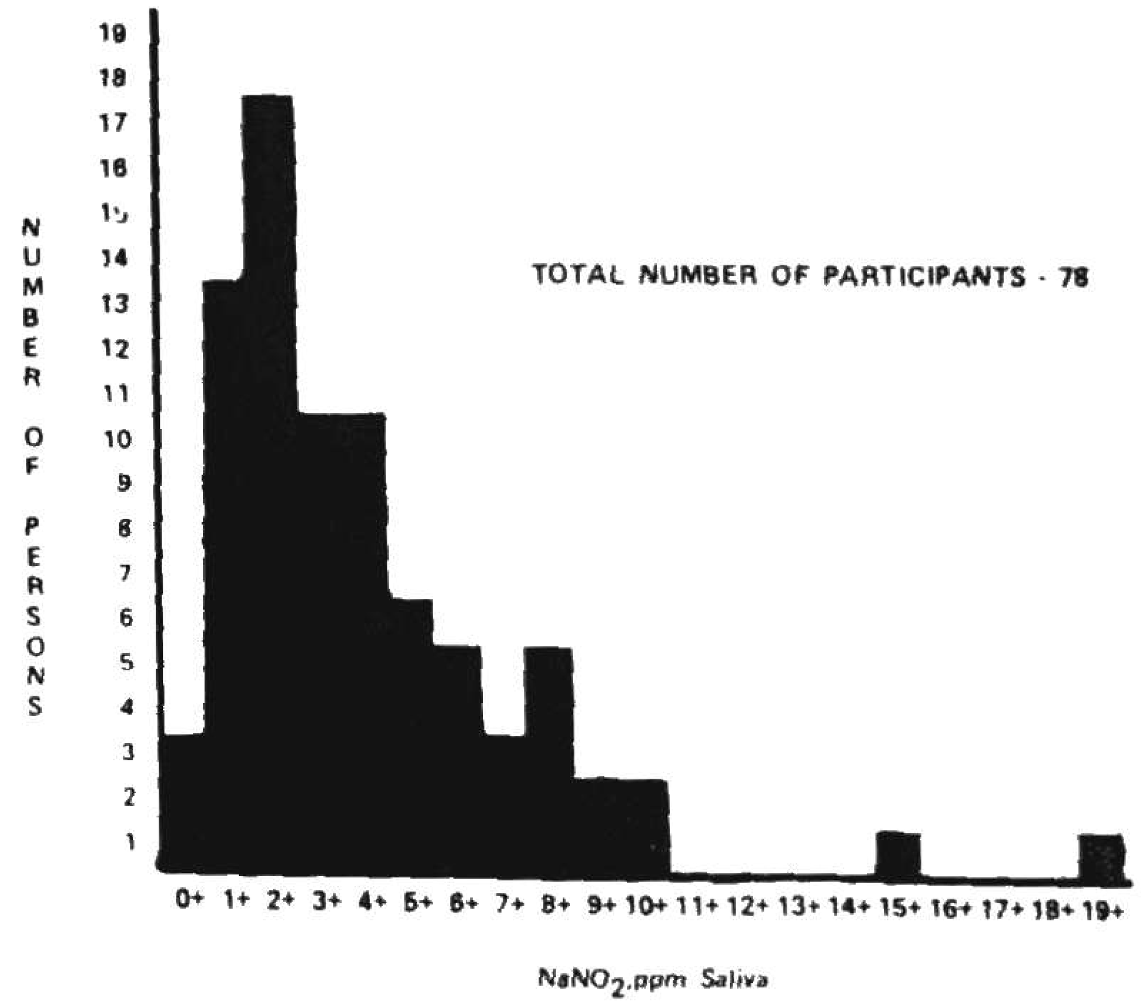

FIG. 2: *Nitrite content of saliva samples from individuals associated with Walters State Community College.*