JOURNAL OF THE TENNESSEE ACADEMY OF SCIENCE

VOLUME 55, NUMBER 1, JANUARY 1980

PROTON-PROTON SCATTERING AT VERY HIGH ENERGIES

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ABSTRACT

A theoretical investigation is carried out on the properties of proton-proton elastic scattering at large energies. It is assumed that both the total and elastic cross sections are rising with energy. Theoretical results are compared to experimental data obtained by Amaldi, Biancastelli, Bosio, Matthiae, Allaby, Bartel, Block, Cocconi, Diddins, Dobinson, Litt and Wetherall.

ASSUMPTIONS AND RESULTS

Proton-proton elastic scattering has been measured at the CERN Intersecting Storage Rings and the results show that both the total and elastic cross sections are rising with energy (Amaldi et al., 1973). These results are consistent with earlier studies based on cosmic-ray data (Yodh, Pal, Trefil; 1972). In this note we consider amplitudes that give rise to increasing cross sections. We assume, first of all, that the effective number of partial waves contributing is

$L = C\sqrt{S}\ (\ln s)^{\gamma}$ where $0 < \gamma < 1$

(Froissart, 1961). This leads to the following bound on the total cross section (Mickens, 1970): $\sigma_T(S) \leq 16\pi C^2\ (\ln s)^{2\gamma}$.

Our second assumption is that $\sigma_T(S)$ has the same energy behavior as the above bound, i.e.,

$$(1)\quad \sigma_T(S) = C_1(\ln S)^{2\gamma}, \quad C_1 \leq 16\pi C^2.$$

We now give the properties of scattering amplitudes which follow from the above two assumptions and compare with the recent experimental data on proton-proton elastic scattering (Amaldi et al., 1973; Bartenev et al., 1972).

Since $\sigma_T(S)$ increases with energy, we must have $\overline{\sigma}_T(S)$ also increasing with energy (Logunov et al., 1973). Thus, we have the Pomeranchuk theorem in the following form,

$$(2)\quad \mathrm{Lim}\ \sigma_T(S)/\overline{\sigma_T(S)} = 1.$$

(Note that Lim means $S \to \infty$.) The following bound exists on the difference of the particle, antiparticle total cross sections (Roy, Singh; 1970),

$$(3)\quad \mathrm{Lim}\ |\overline{\sigma_T(S)} - \sigma_T(S)| \leq B \ln S,$$

where B is a constant. This shows that while $\sigma_T(S)$ and $\overline{\sigma}_T(S)$ may grow like $(\ln s)^{2\gamma}$, their difference can increase at most like $(\ln s)$.

We have the following lower bound on the elastic cross section (Mickens, 1970),

$$(4)\quad \sigma_{el}(S) \geq \left[\frac{C_1^2}{16\pi C^2}\right](\ln S)^{2\gamma}$$

Since $\sigma_{eL}(S)$ is bounded from above by eq. (1), we see that the elastic cross section must have the following energy behavior,

$$(5)\quad \sigma_{el}(S) = C_2(\ln S)^{2\gamma},$$

i.e., the elastic cross section increases at the same rate as the total cross section.

We have the following bound on the phase of the forward scattering amplitude (Mickens, 1970),

$$(6)\quad |\rho(S)| = \left|\frac{\mathrm{Re}\ F(S,0)}{\mathrm{Im}\ F(S,0)}\right| < \left[\frac{C_1}{C^2}\right]^{-\frac{1}{2}}$$

If we denote by $\rho(S)$ and $\overline{\rho(S)}$ the phases for the particle, antiparticle processes, one can show that

$$(7)\quad \mathrm{Lim}\,|\rho(S)| = \mathrm{Lim}\,|\overline{\rho(S)}|.$$

If the symmetric scattering amplitude dominates at high energies, we have (Eden, 1967),

$$(8a)\quad F(S,0) = C_3\, i S\ [\ln(-iS)]^{2\gamma},$$

And

Thus, the real part of the forward amplitude becomes positive for sufficiently high energies.

In the forward direction, we have

$$\frac{d\sigma(S,0)}{dt} = \frac{\sigma_T^2}{16\pi}\left[1 + \rho^2(S)\right].$$

Using eq. (1) and the results given in eq. (7), we obtain

$$(9)\quad \mathrm{Lim}\ \frac{d\overline{\sigma(S,0)}}{dt} = \mathrm{Lim}\ \frac{d\sigma(S,0)}{dt} = \frac{C_1^2(\ln S)^{4\gamma}}{16\pi}\left[1 + \rho^2(S)\right].$$

If we define the diffraction width as (Logunov et al., 1973),

$$(10)\quad \Delta(S) = \sigma_{el}(S)\Big/\frac{d\sigma(S,0)}{dt},$$

we find that,

$$(11)\ \Delta(s) = C_4 / (\ln s)^{2\gamma}.$$

To obtain this result use was made of eqs. (5) and (9). Thus eq. (11) leads us to the conclusion that if the elastic and total cross sections increase with the same energy dependence, the slope of the diffraction peak must increase with energy.

Results

Let us now take a look at the data (Amaldi et al., 1973; Bartenev et al., 1972). Certainly since the elastic and total cross sections are increasing with energy we must have energy behavior that is consistent with both eqs. (1) and (5). However, the data are not over a sufficiently large interval such that γ can be uniquely determined. The values of ρ (s) obtained are consistent with the real part of the forward scattering amplitude becoming positive at ISR energies (Amaldi et al., 1973). Results on the slope of the forward diffraction peak (Bartenev et al., 1972) show that it increases with energy and can be fit approximately by the form, $b(S) = b_0 + b_1 \ln s$. However, the data could probably be fit with other forms for b(S). Since the results given in this note are really asymptotic results, we must be careful at this time of making detailed comparisons between theory and experiment.

Acknowledgement

The results given above are based on an unpublished paper written by the author in 1973. At that time the author received over one hundred requests for copies of the paper.

The author acknowledges all those persons who provided helpful criticism of this paper and hope that they found something of value to their research interests. This research was supported in part by funds from the National Science Foundation and the National Aeronautics and Space Administration.

Literature Cited

Amaldi, U., R. Biancastelli, C. Bosio, G. Matthiae, J. V. Allaby, W. Bartel, M. M. Block, G. Cocconi, A. N. Diddens, R. W. Dobinson, J. Litt and A. M. Wetherell, 1973. Measurement of the Proton-Proton Total Cross Section by Means of Coulomb Scattering at the CERN Intersecting Storage Rings. Phys. Letters 43B: 231-236.

Bartenev, V., A. Kuznetsov, B. Morozov, V. Nikitin, Y. Pilipenko, V. Popov, L. Zolin, R. Carrigan, E. Malamud, R. Yamada, R. L. Cool, K. Goulianas, S. L. Olsen, I-Hung Chiang, A. C. Melissinos and D. Gross, 1972. Small Angle Proton-Proton Scattering from 25 to 200 GeV. Phys. Rev. Letters 29:1755-1758.

Eden, R. J., 1967. *High Energy Collisions of Elementary Particles.* Cambridge University Press. Cambridge, England.

Froissart, M., 1961. Asymptotic Behavior and Subtractions in the Mandelstam Representation. Phys. Rev. 123:1053-1057.

Logunov, A. A., M. A. Mestvirishvili and D. A. Khrustalev, 1973. Restrictions on the Behavior of Elastic and Inelastic Cross Sections at High Energies II. Soviet Journal of Particle and Nuclei 3:261-307.

Mickens, R. E., 1970. Bounds on the Phase of the Forward Scattering Amplitude and the Pomeranchuk Theorem. Lettere al Nuovo Cimento 3:428-432.

Roy, S. M. and V. Singh, 1970. Upper Bounds on Particle-Antiparticle Total Cross Section Differences at High Energies. Phys. Letters 32B: 50-54.

Yodh, G. B., Yash Pal and J. S. Trefil, 1972. Evidence for Rapidly Rising Proton-Proton Total Cross Section from Cosmic-Ray Data. Phys. Rev. Letters 28:1005-1008.

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