Quadraspidiotus juglansregiae (Comstock), 1881 (walnut scale). Collection records: on *Acer* sp. (maple), Knox Co.,* 30 Dec. 1977; on *Cladrastis lutea* (Michx.) (yellowwood), Knox Co., 2 Sept. 1977; on *Cornus florida* L. (flowering dogwood), Blount Co.,* 17 Aug. 1976; Knox Co., 4 Apr. 1975; on *Fraxinus americana* L. (white ash), Knox. Co., 16 Aug. 1975; on *Ilex crenata* Thunb. (Japanese holly), Shelby Co., 17 Aug. 1977; on *Liriodendron tulipifera* L. (yellow-poplar), Knox Co., 25 Jun. 1976; on *Quercus palustris* Muenchh. (pin oak), Knox Co., 6 Aug. 1976.

Quadraspidiotus perniciosus (Comstock), 1881 (San Jose scale). Collection records: on *Betula pendula* Roth. (European white birch), Knox Co., 27 Dec. 1977; on *Liriodendron tulipifera* L. (yellow-poplar), Knox Co., 26 Jun. 1975; on *Malus hybrida* Desf. (flowering crab apple), Knox Co., 7 Aug. 1975; on *Malus pumila* Mill. (apple), Cocke Co.,* 23 Aug. 1977; Davidson Co., 29 Aug. 1976; Polk Co., 6 Oct. 1976; Sullivan Co., 11 Apr. 1977; on *Prunus persica* (L.) (peach), Sullivan Co., 11 Apr. 1977; on *Pyrus* sp., Morgan Co.,* 12 Jul. 1976.

Unaspis euonymi (Comstock), 1881 (euonymus scale). Collection records: on *Euonymus alata* (Thunb.) (winged euonymus), Blount Co.,* 18 May 1977; Campbell Co.,* 31 Aug. 1976; Knox Co.,* 22 Aug. 1976.

Velataspis dentata (Hoke), 1921, (dentata scale). Collection records: on *Cornus florida* L. (flowering dogwood), Blount Co.,* 7 Aug. 1977.

Conclusions

In this study, 74 species on 206 hosts were collected from November 1974 through December 1977. Twenty-six species were recorded for the first time in Tennessee, and 142 new county records were obtained.

Several species are host specific at least to the plant family level. Oligophagous species obtained were: *Asterolecanium minus, Kermes galliformis, K. pubescens, Lecanium quercifex* and *Protodiaspis varus* on Fagaceae; *Diaspis echinocacti* and *Eriococcus coccineus* on Cactaceae; *Diaspidiotus liquidambaris* on Hamamelidaceae; *Chionaspis pinifoliae, Matsucoccus gallicolus, Toumeyella parvicornus* and *T. pini* on Pinaceae; and *Neolecanium cornuparvum* on Magnoliaceae. All other species collected were polyphagous (eg. *Lepidosaphes ulmi* and *Planococcus citri*) and have been recorded on hundreds of hosts.

Acknowledgement

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A MATHEMATICAL ANALYSIS OF WORLD CHESS CHAMPIONSHIP SYSTEMS

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Abstract

In 1975 Robert Fischer resigned his title as Chess Champion of the World in a dispute with FIDE, the world chess organization, over the match rules for his upcoming title defense. Proceeding from the premise that it is the object of every chess match to determine which of two players is the better, several match systems are analyzed. The systems are compared in terms of the match winning probabilities of two hypothetical players, Alpha and Beta, under each set of match rules. Formulas for these probabilities are derived in terms of the symbols, A, B, and D which represent the per game probabilities of a win by Alpha, a win by Beta, and a drawn game, respectively. The systems are then compared to determine which one most favors the better player. The match systems treated include the 1972 match rules, in effect when Fischer won his title; the Fischer system, proposed in 1975; and the 1978 system, used in the recent world championship match between Anatoly Karpov and Victor Korchnoi.

Introduction

In 1975 Robert Fischer resigned his title as World Chess Champion in a dispute with FIDE, the world chess organization, over the match rules for his title defense. When Fischer won the title the match rules were as follows:

(1) The match consisted of a maximum of twenty-four games with 1 point being awarded to the winner of a game and ½ point to each player for a drawn game.

(2) The first player to score more than 12 points wins the match and the title, but in the event of a drawn match (12-12) the champion retains the title.

Fischer proposed that this system be changed as follows:

(1) The first player to score 10 wins, wins the match and title.

(2) Draws do not count and, therefore, there is no fixed limit on the number of games played.

(3) Should the score ever reach 9-9, the match is declared a draw and the champion retains his title.

FIDE agreed to all of Fischer's proposals except the last. Opponents argued that this clause gave the Champion an unfair advantage, since a challenger would have to win by at least a two game margin (10-8), whereas a one game margin would be sufficient under the old system. Supporters claimed that the above argument did not take into account the difference between a limited system (24 games) versus the unlimited system proposed by Fischer. Both systems give an edge to the champion, since he can retain his title either by winning or drawing the match.

In the most recent world championship match a Fischer type system was employed in which the first player to attain six wins was adjudged the victor. There was no provision for a drawn match, but there was a requirement for a second match within a year if the champion should lose. In the analysis which follows all of these systems will be compared.

METHOD

In this analysis a chess match is regarded as a sampling of the play of two contestants in an effort to determine which is the better player. The better player is defined as the one who would win the most games if a large number were to be played. Match systems are viewed as prescriptions for different sampling techniques and the problem is to access the relative reliabilities of those techniques.

To accomplish the task outlined above, some assumptions must be made about the nature of the competition in a match. The basic assumption made here is that if a large number of games could be played under the same conditions which exist during a given match, then the relative frequencies of wins, losses, and draws would approach stable limits. Consider, therefore, two hypothetical players, Alpha and Beta, and let the symbols, A, B, and D stand for the relative frequencies of wins by Alpha, wins by Beta, and drawn games, respectively. Formulas will be derived in terms of these symbols for the match winning probabilities of Alpha and Beta under each of the match systems, each game being considered an independent trial. The formulas are then used to determine if the better player has a higher match winning probability under one system than the others.

Since the assumptions made above can never be proven, they are open to question. It might be felt that each game should not be treated as an independent trial and that a match is a stochastic process in which the probabilities vary from trial to trial. It is then necessary to make further assumptions about the nature of this process. Such an approach is, of course, possible and provides an interesting area for further investigation. The point of view adopted here, however, is that the fewer the number of unverifiable assumptions, the better.

ANALYSIS

Treating the Fischer system first, Alpha's match winning probability will be calculated. The following facts concerning this case are noted: (1) Alpha must win 10 games, (2) Beta can win any number of games between 0 and 8, (3) there can be any number of draws in between, and (4) the last game of the match will be a win for Alpha. The problem is to ascertain all the different ways Alpha could possibly win the match, the probability of each way, and sum all the probabilities. For example, Alpha might win by a score of 10-7 with no draws. The probability of this is $A^{10}B^7$. There are, however, many different ways this could happen, each way having this same probability. Two such possibilities are illustrated on the next page.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	A	A	A	A	A	A	A	B	A	B	B	B	B	B	B	A

The numbers refer to the order in which the games are played, and the letters indicate the winner of particular games. The only difference in the two illustrations is the interchange of Alpha and Beta as the winners of games 9 and 10. Ascertaining how many different ways Alpha could compile a 10-7 score with no draws is equivalent to determining the number of different ways 9 A's and seven B's can be ordered. The reason for stating 9 A's instead of 10 A's is that in every ordering the last game must have Alpha as its winner, since the match always ends upon completion of the 10th win. It follows that the number of different ways a 10-7 score could occur with no draws is 16!/9!7!, and the probability of this event is $(16!/9!7!)A^{10}B^7$. It now remains to consider the probability of Alpha's winning by a score of 10-7 with one draw, two draws, and so on without end. The following table illustrates some of these possibilities together with the probability of each.

TABLE I: *Events producing a 10-7 score.*

Wins	Losses	Draws	Probability
10	7	0	$(19!/9!7!)A^{10}B^7$
10	7	1	$(17!/9!7!1!)A^{10}B^7D$
10	7	2	$(18!/9!7!2!)A^{10}B^7D^2$
10	7	3	$(19!/9!7!3!)A^{10}B^7D^3$

Alpha's total probability of winning by 10-7 is an infinite sum of terms such as those illustrated in the table. Letting P(10, 7) stand for this probability, we have

(1) $P(10, 7) = (16!/9!7!)A^{10}B^7 [1 + 17D/1! + (17)(18) D^2/2! + (17)(18)(19) D^3/3! + \ldots\ldots]$

The infinite sum within the brackets is just the binomial expansion of $(1\text{-}D)^{-17}$, and since $1 - D = A + B$, it follows that

(2) $P(10, 7) = C(16, 7)\, A^{10}B^7 (A+B)^{-17}$

Letting $R = A/B$, we have

(3) $P(10, 7) = C(16, 7)\, R^{10} (1+R)^{-17}$

and, in general

(4) $P(10, N) = C(10+N-1, N)\, R^{10} (1+R)^{-(10+N)}$

Using PF (A) to stand for Alpha's match winning probability under the Fischer system, we have

(5) $PF(A) = \sum_{N=0}^{8} P(10, N)$

Beta's match winning probability can be obtained by interchanging the symbols A and B, and the probability of a drawn match is

(6) $PF(D) = 2C(17, 8)\, R^9 (1+R)^{-18}$

C (17, 8) represents the number of different ways one of the players could achieve the last win to make the score 9-9. Since either player could be the one to do this, the total number of ways the match could end in a draw is twice this factor.

In 1975 FIDE agreed to all of Fischer's proposals except the draw clause. Letting P75(A) stand for Alpha's probability of winning a match in this system, we have from the previous discussion that

(7) $P75(A) = \sum_{N=0}^{9} P(10, N)$

This differs from equation (5) only in the range of the summation index.

In the 1978 World Championship Match the victor was the first player to score six wins and with no provision for a drawn match. Alpha's probability of winning a single match under these rules is given by

(8) $P6(A) = \sum_{N=0}^{5} P(6, N)$

According to the 1978 rules, however, the challenger, if victorious in the first match, would have to play a return match under the same rules. He thus had to win back-to-back matches and that probability is the square of the result given in equation (8). Calling this probability P78 (A), we have

(9) $P78(A) = [P6(A)]^2$

It should be noted that in all the foregoing systems the match winning probabilities depend only on the ratio of A to B, the draw rate being irrelevant.

In calculating Alpha's match winning probability under the old match rules, it should be noted that there are two possible scores for the victor—13 points or 12½ points. Beginning with the 13 point case, it is important to consider the following:

(1) the last game of the match is a win for the match victor, (2) there are an even number of draws, if any, and (3) the loser's score is always an integral number of points.

The final score in this case will always be of the form 13-N, where N is an integer between 0 and 11. Four of these possible match scores, together with all the possible combinations of wins, losses, and draws producing them, are listed in the table below.

TABLE II: *Events producing a winning score of 13 points.*

Score	W	L	D	Probability
13-0	13	0	0	$(12!/12!0!0!)A^{13}B^0D^0$
13-1	13	1	0	$(13!/12!1!0!)A^{13}B^1D^0$
	12	0	2	$(13!/11!0!2!)A^{12}B^0D^2$
13-2	13	2	0	$(14!/12!2!0!)A^{13}B^2D^0$
	12	1	2	$(14!/11!1!2!)A^{12}B^1D^2$
	11	0	4	$(14!/10!0!4!)A^{11}B^0D^4$
13-3	13	3	0	$(15!/12!3!0!)A^{13}B^3D^0$
	12	2	2	$(15!/11!2!2!)A^{12}B^2D^2$
	11	1	4	$(15!/10!1!4!)A^{11}B^1D^4$
	10	0	6	$(15!/9!0!6!)A^{10}B^0D^6$

As an example, consider the case in which Alpha scores 11 wins, 1 loss, and 4 draws. The number of ways this can occur is equal to the number of different ways of ordering 10 A's, 1 B, and 4 D's. This is 15!/10!1!4! and the probability of this event is (15!/10!1!4!)-$A^{11}B^1D^4$. Alpha's total probability of winning with a final score of 13 points is the sum of all the probabilities in the table plus those for the cases not listed. Letting P (13) stand for this probability, the result is

(10) $P(13) = \sum_{j=13}^{24} (j\text{-}1)! \sum_{i=0}^{j-13} [1/(12\text{-}i)!(j\text{-}13\text{-}i)!(2i)!] \times A^{(13-i)} B^{(j-13-i)} D^{(2i)}$

In treating the 12½ point case the following facts are noted:

(1) there will always be an odd number of draws, (2) there will be at least one draw, and (3) the final game of the match will be a draw or a win for the match victor.

Table III lists four possible match scores for this case, together with all the combinations of wins, losses, and draws producing them.

TABLE III: *Events producing a winning score of 12½ points.*

Score	W	L	D	Probability
12½-½	12	0	1	$(12!/11!0!1!+12!/12!0!0!)A^{12}B^0D^1$
12½-1½	12	1	1	$(13!/11!1!1!+13!/12!1!0!)A^{12}B^1D^1$
	11	0	3	$(13!/10!0!3!+13!/11!0!2!)A^{11}B^0D^3$
12½-2½	12	2	1	$(14!/11!2!1!+14!/12!2!0!)A^{12}B^2D^1$
	11	1	3	$(14!/10!1!3!+14!/11!1!2!)A^{11}B^1D^3$
	10	0	5	$(14!/9!0!5! +14!/10!0!4!)A^{10}B^0D^5$
12½-3½	12	3	1	$(15!/11!3!1!+15!/12!3!0!)A^{12}B^3D^1$
	11	2	3	$(15!/10!2!3!+15!/11!2!2!)A^{11}B^2D^3$
	10	1	5	$(15!/9!1!5! +15!/10!1!4!)A^{10}B^1D^5$
	9	0	7	$(15!/8!0!7! +15!/9!0!6!)A^9B^0D^7$

Consider, for example, the case where Alpha scores 11 wins, 2 losses and 3 draws. There are 15!/10!2!3! different ways this could occur with Alpha winning the final game and 15!/11!2!2! different ways it could occur with the last game of the match ending in a draw. Therefore, the total number of ways this result could occur is the sum of these two and the probability of the event is as listed in the table. Alpha's total probability of winning the match with a final score of 12½ points is the sum of all the probabilities in the table plus those for the cases not listed. Letting P(12½) stand for this sum, we have

(11) $P(12\tfrac{1}{2}) = \sum_{j=13}^{24} (j\text{-}1)! \sum_{i=0}^{j-13} \left[\frac{1}{(12\text{-}i)!(j\text{-}13\text{-}i)!(2i)!} + \frac{1}{(11\text{-}1)!(j\text{-}13\text{-}2)!2i+1)!} \right] A^{(12-i)} B^{(j-13-i)}\ D^{(2i+1)}$

Therefore, if we let P72(A) stand for Alpha's total probability of winning a match under these rules, we have

12) $P72(A) = P(13) + P(12\tfrac{1}{2})$

In a similar manner it can be shown that the probability of a drawn match under this system is

$$(13)\quad P72(D) = \sum_{i=0}^{12} [24!/(i!)^2(24-2i)!]\, A^iB^iD^{(24-2i)}$$

RESULTS

The systems are now compared by calculating Alpha's match winning probabilities for various values of A, B, and D. The computations were made from the formulas by a DEC PDP-11 computer. As an internal check on the correctness of the formulas, both player's win probabilities, as well as the probability of a drawn match (for those systems where applicable) were calculated for each set of values of A, B, and D. The sum of these probabilities was always one (1), as, of course, it should be. In all cases treated, Alpha is assumed to be the better player (A>B), and is also assumed to be the challenger. His match winning probabilities are listed in Table IV.

TABLE IV: *Alpha's match winning probabilities.*

Match System	Alpha's Relative Playing Strength Advantage				
	51-49	52-48	53-47	54-46	55-45
P75 (A)	.535197	.570142	.604586	.638292	.671036
P6 (A)	.527052	.553997	.580726	.607135	.633123
PF (A)	.440947	.475077	.509426	.543762	.577849
P72 (A)					
D=.5	.470327	.498087	.525874	.553567	.581043
D=.6	.460416	.485204	.510061	.534903	.559642
D=.7	.446743	.468133	.489622	.511156	.532679
D=.8	.425033	.442350	.459775	.477282	.494843
P78 (A)	.277784	.306912	.337242	.368613	.400844

The structure of the table can be understood by considering Alpha's match winning probabilities in the first column for the 1972 system. The values of A, B, and D used in the calculations of the four values found there are (.255, .245, .5), (.204, .196, .6), (.153, .147, .7), and (.102, .098, .8). The ratio of A to B is 51-49 in every case. Since Alpha's probabilities for the other systems depend only on the ratio of A to B, it does not matter which of the above is used in those calculations. For this reason, the cases are categorized by listing the ratio of A to B rather than their specific values. These whole number ratios may be interpreted as meaning that out of a hundred decisive games Alpha's edge in playing strength, expressed in games won and lost, is as shown.

The table reveals that Alpha is most favored by the ten win system, P75, which allows for no possibility of a drawn match and no requirement for a return match if the champion loses. The next best is the similar six win system. The worst is the six win system with the provision for a return match—the system used in the 1978 title defense. The reason, of course, is that if P is the probability of winning one match under a given system, P^2 is the probability of winning two, and since $P<1$, $P^2<P$. It is not too surprising, therefore, that this system is the only one in the table for which Alpha, though the better player, is actually the underdog for all cases treated. He does not become the favorite until his edge is 58-42, and even then his probability of winning both matches is only .500699. It is obvious that the addition of a return match clause to any of the other systems would produce similar disparities between playing strength and title winning chances.

In comparing the two remaining systems in the table, the Fischer system and the 1972 system, it is noted that the latter is to be preferred if the draw rate is 50% or less. This line of the table has been included for completeness, but is of little practical significance, since no championship match has had that small a percentage of draws. Draw rates in the sixty to eighty percent range are more likely. For a draw rate of sixty percent the two systems are about equal when the playing strengths are 53-47. The Fischer system is to be preferred, even at this level, for higher draw rates and is the clear choice at higher playing strength levels. The 1972 system is for choice if Alpha's advantage is less than 53-47 and the draw rate no greater than 60%. Even so, for levels less than 53-47 Alpha's probability under either system is less than .5, and, therefore, it is more likely that he will either lose or draw the match than win it. Only at the 53-47 level does Alpha become the over all favorite, and from this point on the Fischer system is the better of the two.

From the preceding discussion it is not hard to see that better systems than the ones treated here can be proposed. For example, a single match requiring twelve wins for victory with no provision for a drawn match and no return match clause would be better than the similar ten win system, which was the best of those compared in Table IV. Obviously, by requiring greater numbers of wins for victory one can increase the probability that the ultimate match victor is, indeed, the better player. It is equally obvious that practical considerations must take precedence over this process. In the 1978 championship match, for example, thirty-two games were played before the six win limit was reached. This was the longest championship match since Capablanca-Alekhine in 1927. While experience with match systems of this type is limited, increasing the win limit beyond six games would certainly tend to make for longer matches.

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