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A DESCRIPTION OF THE PLEISTOCENE FAUNAL REMAINS RECOVERED FROM FINGER QUARRY, BLOUNT COUNTY, TENNESSEE

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ABSTRACT

The skeletal remains of at least 16 species of vertebrates were recovered from Finger Quarry in Blount County, Tennessee. At least five of the 16 species represented in the faunal sample are extinct, North American, Pleistocene animals (*Dasypus bellus, Felis onca, Equus*, sp., *Tapirus* cf. *veroensis*, and *Mylohyus* sp.). Descriptions are provided for elements of both the extant and the extinct vertebrates recovered from the quarry.

INTRODUCTION

The assemblage of Pleistocene skeletal remains described in this report was recovered from Finger Quarry in Blount County, Tennessee, by Mr. John J. Craig and several of his employees. Finger Quarry is located at 35°45′56″ north latitude, and 84°7′7″ west longitude (U.S.G.S. Louisville, Tennessee Quadrangle, 7.5′ series, 1968). This places the quarry just slightly outside of and to the northeast of Friendsville, Tennessee, near Vinegar Valley Road. The skeletal remains were removed from the marble quarry in ca. 1968-69, but have not been described until now. An earlier find of fossil materials was made at another Blount County quarry operated by Mr. Craig and should not be confused with this separate discovery. Corgin (1976) recorded the earlier find in his book on Tennessee vertebrate fossils.

DESCRIPTION OF SITE

At Finger Quarry, the Pleistocene skeletal materials were found in the clay fill of a solution pocket in the Ordovician Holston marble formation (Stuart Maher, Tennessee Division of Geology, personal communication). The solution pocket containing the fossil materials was a nearly vertical fissure approximately 30 feet long and between 12 and 18 inches wide. Bone fragments and teeth were found mixed throughout the clay fill of the solution pocket, starting at approximately 30 to 35 feet below the present ground surface. No stratigraphy was noted within the clay fill; all osteological materials were removed by hand from the clay matrix. The solution pocket no longer exists since the clay and the marble surrounding it were removed.

RESULTS

Most of the material in this faunal sample consists of teeth (of 88 identifiable skeletal elements, 60 are teeth). Remains of at least 15 species of mammals and one avian species, representing 14 families, comprised the skeletal collection (Table I). In addition, there were also 30 indeterminate mammal and three indeterminate bird bone fragments present. Nearly all of the bone remains as well as the roots of the teeth were severely gnawed by rodents.

TABLE 1: *Fauna Identified from Finger Quarry, Blount County, Tennessee*

Species	No. of Pieces	MNI
Aves		
Tetraonidae		
cf. *Bonasa umbellus* (Linnaeus) Ruffed Grouse	1	1
Mammalia		
Didelphidae		
Didelphis marsupialis Linnaeus—Opossum	4	1
Dasypodidae		
Dasypus bellus Simpson—"Beautiful Armadillo"	6	1
Leporidae		
Sylvilagus and/or *Lepus*—Cottontail and/or Snowshoe Hare	2	1
Sciuridae		
Marmota monax (Linnaeus)—Woodchuck	7	2
Cricetidae		
Neotoma floridana (Ord)—Eastern Woodrat	4	2
Canidae		
Urocyon cinereoargenteus (Schreber)—Gray Fox	1	1
Canis sp.—Indeterminate Canid	1	1
Ursidae		
Ursus americanus Pallas—Black Bear	10	1
Procyonidae		
Procyon lotor (Linnaeus)—Raccoon	1	1
Felidae		
Felis onca Linnaeus—Jaguar	1	1
Equidae		
Equus sp.—Horse	16	4
Tapiridae		
Tapirus cf. *veroensis* Sellards—Tapir	5	2
Tayassuidae		
Mylohyus sp.—Long-nosed Peccary	4	1
Cervidae		
Odocoileus virginianus (Zimmerman)—White-tailed Deer	14	4
Cervid sp.	1	1

It is difficult to determine the contemporaneity of the materials found in the fissure at Finger Quarry. The fissure may have remained open for some time, thus allowing for a slow accumulation of osetological

materials, or it may have filled rapidly, with the materials being deposited during a very short time period. Faunal materials within the solution pocket may either have washed into the fissure, or may represent, in part, hoard objects accumulated by woodrats. That woodrats were in the vicinity is attested to by the presence of their remains in the deposit. Accumulation by either means, natural wash or woodrats, does not preclude the possibility that older faunal materials may have been mixed with those from a more recent time period.

Because of the small size of the collection and the unknown means or rate of deposition, it is not possible to determine whether or not the species present coexisted during the time period of bone accumulation. The collection is, however, significant in that it documents range extensions in Tennessee for a number of extinct Pleistocene species.

All of the faunal materials described in this report, with the exception of one deer and three horse elements, are in the possession of James L. Craig, son of John J. Craig, owner of Craig Marble Company. These four skeletal elements were retained by John A. Coffey, Sr., an employee of the Craig Marble Company.

Accounts of Species

Class Aves

Order Galliformes

Family Tetraonidae

cf. *Bonasa umbellus* (Linnaeus)—Ruffed Grouse

Material: 1 proximal half of a left humerus, MNI (Minimum Number of Individuals): 1.

Remarks: Enough of this fragmentary humerus was present to definitely assign it to the family Tetraonidae, but it could only be tentatively identified as ruffed grouse. Ruffed grouse are currently part of the fauna in East Tennessee (White and Dimmick, 1979).

Class Mammalia

Order Marsupialia

Family Didelphidae

Didelphis marsupialis Linnaeus—Opossum

Material:1 left P_3; 2 left M_1's or M_2's; 1 left M_3 or M_4. MNI: 1.

Remarks: All of the teeth in this sample show very little wear and are probably from a single young individual. Because of the close similarity in size and configuration of the M_1 and M_2, and respectively the M_3 and M_4, no further attempt was made to specifically designate each as to position.

Since all the bone materials in the fissure are not necessarily contemporaneous, it cannot definitely be stated that the opossum recovered at the site coexisted with the extinct giant armadillo, tapir, jaguar, or peccary. Guilday, *et al.* (1978) note that opossum remains have been found in good context with Pleistocene materials only as far north as Bartow County, Georgia. In late prehistoric archeological sites, however, opossum remains are commonly found as far north as west-central West Virginia.

Order Edentata

Family Dasypodidae

Dasypus bellus Simpson—"Beautiful Armadillo"

Material: 6 fragmentary dermal scutes. MNI: 1.

Remarks: In addition to the Finger Quarry material, giant armadillo remains have been recovered from only three other Tennessee sites, Robinson Cave (Guilday and McCrady, 1966), Baker Bluff Cave (Guilday, *et al.*, 1978), and Cheek Bend Cave (Klippel, n.d.). The former occurrence of armadillos in Tennessee, and as far north as West Virginia, suggests that the climate at some period during the late Pleistocene was much milder than at present (Guilday and McCrady, 1966).

Order Lagomorpha

Family Leporidae

Sylvilagus and/or *Lepus*—Cottontail? and/or Snowshoe Hare?

Remarks: Based upon a fragmentary mandible and an isolated mandibular incisor, it was not possible to more specifically identify these Leporid remains. While *Sylvilagus floridanus* is currently found in the vicinity of the quarry, Burt and Grossenheider (1976) feel that *Lepus americanus* may occur in Tennessee today only in the higher elevations of the Appalachian Mountains. If this is correct, it is quite possible that *L. americanus* inhabited lower elevations within the adjacent Ridge and Valley Provence during the Pleistocene. Guilday, *et al.* (1977) also report the occurrence of either *Sylvilagus* or *Lepus* from Baker Bluff Cave deposit, Sullivan County, in northeastern Tennessee.

Order Rodentia

Family Sciuridae

Marmota monax (Linnaeus)—Woodchuck

Material: 3 upper incisors; 3 lower incisors; 1 incisor fragment. MNI: 2.

Remarks: Because of their burrowing habits, woodchuck remains are common in most Pleistocene deposits in the eastern United States. It is thought here, however, because of the deep, rather narrow nature of the Finger Quarry solution cavity, that the woodchuck remains may have washed into the deposit or been carried there by woodrats. An interesting dental anomaly was noted in this sample of woodchuck incisors. Most frequently, woodchuck incisors are white in color, but four of the six nearly complete incisors in this sample are yellow.

Family Cricetidae

Neotoma floridana (Ord)—Eastern Woodrat

Material: 2 right anterior mandible fragments, each with incisor present; 1 right proximal femur; 1 right innominate. MNI: 2.

Remarks: The woodrat is still known to occur in the vicinity of the site today. Although its modern distribution is primarily temperate in nature, Guilday, *et al.* (1977) suggest that the woodrat may also have been part of the boreal Pleistocene fauna during the Wisconsinan glaciation. It is quite possible that the woodrat may have been the principal rodent responsible for gnawing away much of the bone found at Finger Quarry.

Order Carnivora

Family Canidae

Urocyon cinereoargenteus (Schreber)—Gray Fox

Material: 1 right M_1. MNI: 1.

Remarks: The fox carnasial from the Finger Quarry deposit compared very favorably in size and cusp configuration with those from a series of *U. cinereoargenteus* recently collected in Tennessee and Illinois. The right M_1 recovered from Finger Quarry had a length of 12.9 mm and a width of 5.8 mm. In examining a sample of seven modern gray foxes, the observed range of right M_1 length was 11.7-14.0 mm; the observed range of width was 5.0-6.3 mm. The tooth was from an adult animal and is slightly worn. Currently, the gray fox is found over most of Tennessee.

Canis sp.—Indeterminate Canid

Material: 1 left P^4. MNI: 1.

Remarks: A left P^4 from an indeterminatte species of *Canis* was also recovered from Finger Quarry. This specimen was from an immature animal, the roots having not yet fully developed. The tooth compares favorably with the same tooth from a modern dog, the size of a beagle. Crown length is 15.1 mm, width is 7.5 mm.

Family Ursidae

Ursus americanus Pallas—Black Bear

Material: 1 right M^2; 1 left M_1; 1 left M_2; 1 left M_3; 1 right lower C; 3 C fragments; 1 second phalanx; cf. 1 left radius shaft. MNI: 1.

Remarks: All the measureable black bear teeth found at Finger Quarry compared closely in size with those of modern local black bears (Table 2). In addition to the teeth and a second phalanx, another bone element was recovered which may be black bear. Although the element is badly eroded and rodent gnawed, it most closely approximates the left radius shaft of a bear. Bear skeletal remains are reported to be some of the most commonly found Pleistocene fossils (Guilday, *et al.*, 1978).

TABLE 2: *Dentition measurements (mm) of Ursus americanus from Finger Quarry compared with bears taken in the Great Smoky Mountains, Tennessee (Coll. UTK, Forestry/Wildl. Mgt. Dept.)*

	Finger Quarry Bear		Modern Black Bears 6 males, East Tennessee		
Tooth	N	Measurement	N	O.R.	$\bar{X}$
M^2 length	1	31.7	6	26.0-29.8	27.9
M^2 width	1	16.4	6	14.7-16.5	15.9
M_1 length	1	20.3	6	18.4-20.4	19.4
M_1 width	1	9.5	6	9.3-10.5	9.7
M_2 length	1	19.1	6	20.5-22.7	21.3
M_2 width	1	12.1	6	11.9-14.1	12.9
M_3 length	1	17.2	6	14.9-17.2	16.2
M_3 width	1	12.5	6	11.6-13.5	12.4

Family Procyonidae

Procyon lotor (Linnaeus)—Raccoon

Material: 1 metapodial. MNI: 1.

Remarks: The metapodial was from a large adult animal.

Family Felidae

Felis onca Linnaeus—Jaguar

Material: 1 P_4.MNI: 1.

Remarks: A single, incomplete jaguar fourth mandibular premolar was recovered from the clay pocket at Finger Quarry. This tooth is most probably from *F. o. augusta*, the extinct late Pleistocene jaguar sub-species known to have inhabited Tennessee. Additional jaguar (*F. o. augusta*) remains have been found at six other locations in Tennessee (all in caves) and are described in reports by Guilday and McGinnis (1972) and Guilday, *et al.* (1978).

Perissodactyla

Family Equidae

Equus sp.—Horse

Material: 1 right dP_1; 2 right $dP_{2\text{-}4}$ or $dP_{3\text{-}4}$; 1 left P^2; 2 right $P_{2\text{-}4}$; 1 right M^3; 1 left M^3; 2 right maxillary premolar/molars; 2 left mandibular premolar/molars; 3 tooth fragments; 1 first phalanx. MNI: 4.

Remarks: In examining the horse teeth, the author found that it was not possible to identify the exact placement of each tooth in the dental arcade. Only the second adult premolars and the fourth molars, in addition to the deciduous first premolar, were easily recognizable. It is relatively easy to differentiate horse maxillary check teeth from mandibular check teeth and to determine whether they are from the right or left side, but it is extremely difficult to specifically identify which molar or premolar one is examining.

Maher (n.d.), in his unpublished list of nearby Craig Quarry skeletal remains, referred to his horse material as *Equus complicatus* one of the more common late Pleistocene, North American equine species. Making specific identification on the basis of isolated teeth is rather dubious since Corgin (1976) states that there may have been at least three different horse species (*E. leidyi, E. complicatus*, and *E. conversidens*) present in Tennesseee during the Pleistocene.

Family Tapiridae

Tapirus cf. *veroensis* Sellards—Tapir

Material: 1 left p^1; 1 right p^1; 3 tooth crown fragments. MNI: 1.

Remarks: Both of the recovered left and right $P^{1/s}$ are from young individuals and neither displays much occlusal wear. Because of differences in their size, it is obvious that these two teeth are from separate animals. Measurements of both teeth suggest that they at least tentatively be placed in the species *T. veroensis* (Lundelius and Slaughter, 1976) (Table 3). Tapir remains are commonly found in Pleistocene fossil deposits in the Eastern United States; the animals distribution extended from Pennsylvania to Florida. Bogan, *et al.* (1980) summarize the tapir remains found at 10 additional localities within Tennessee.

TABLE 3: *Measurements (mm) of Tapir Teeth from Finger Quarry*

Finger Quarry Tapir			*Tapirus veroensis* (Lundelius and Slaughter 1976)		
Tooth	N	Measurement	Tooth	O.R.	$\bar{X}$
left p^1 length	1	19.4	p^1 length	17.5-20.8	18.9
left p^1 width	1	18.0	p^1 width	14.9-18.6	17.3
right p^1 length	1	17.4			
right p^1 width	1	15.8			

Order Artiodactyla

Family Tayassuidae

Mylohyus sp.—Long-nosed Peccary

Material: 1 left mandible fragment with P_4M_1; 1 right M_3; 2 left $P^{3/s}$ or $P^{4/s}$. MNI: 1.

Remarks: There is some confusion as to whether both species of *Mylohyus* (*M. nasutus* and *M. fossilis*) are found within Tennessee. Lundelius (1960) feels that *M. fossilis* was an East Coast form ranging from Florida to Maryland, while *M. nasutus* is believed to have had a more westerly distribution. Corgin (1976) is of the opinion that both of the species occurred within Tennessee. Insufficient *Mylohyus* skeletal remains recovered at Finger Quarry prohibit positive species identification. Measurements of the complete teeth are presented here, however, for future reference (Table 4).

TABLE 4: *Finger Quarry Mylohyus Tooth Measurements (mm).*

Tooth	N	Measurement
left P_4 length	1	14.3
left P_4 AW*	1	12.5
left P_4 PW*	1	12.4
left M_1 length	1	15.8
left M_1 AW	1	13.5
left M_1 PW	1	13.1
right M_3 length	1	21.9
right M_3 AW	1	13.0
right M_3 PW	1	11.6

*AW=Anterior Width
*PW=Posterior Width

Family Cervidae

Odocoileus virginianus (Zimmerman)—White-tailed Deer

Material: 2 left $dP_{3\text{-}4}$; 1 left P_2; 1 right M_3; 1 left M_3; 2 left $P^{4/s}$, 1 left M^1; 2 right $M^{1/s}$; 1 right scapula; 2 right metatarsal fragments; 1 left humerus shaft. MNI: 4.

Remarks: The skeletal remains of *Odocoileus virginianus* from the site closely approximate in size those of modern deer from Tennessee.

Cervid sp.

Material: 1 right mandibular molar fragment. MNI: 1.

Remarks: In addition to the *Odocoileus* material, another cervid element from a different species was found at the site. This piece consists of one half of a right mandibular molar from an animal the size of an elk, *Cervus elephas*. The tooth was not complete enough to warrant a more exact identification.

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AN OSCILLATION THEOREM FOR A SECOND ORDER SUBLINEAR DIFFERENTIAL EQUATION WITH DELAY

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Abstract

Conditions are given which are necessary and sufficient for oscillation of second order sublinear differential equations with delay including functional equations.

Introduction

This paper gives necessary and sufficient conditions for oscillation of delay equations of the form

$$y''(t) + f(t, y(t), y(g(t))) = 0 \qquad (1)$$

and functional delay equations

$$y''(t) + F(t, y_t) = 0 \qquad (2)$$

In (1) f (t, u, v) satisfies the usual continuity and sign conditions (Erbe 1973; Wong, 1971) and $g(t) \leq t$, g(t) continuous, $\lim_{t\to\infty} g(t) = \infty$.

In (2) F (t, y_t) again satisfiies the usual continuity and sign conditions (Burkowski, 1971; Siam 1871) where y_t is the restriction of y (.) to [t-m, t]. We will further follow the notation of Burkowski.

Preliminary Remarks

The following lemma will play a useful role in the two main theorems presented in this paper.

LEMMA. Suppose $y(t) > 0$, $y'(t) \geq 0$, $y''(t) \leq 0$ or $y(t) < 0$, $y'(t) \leq 0$, $y''(t) \geq 0$ for $t \geq t_0$. For $d>0$ there exists a $t_1 > t_0$ such that

(i) $\frac{y(t)}{y(t)} \leq \frac{1+d}{t}$ for $t \geq t_1$ and

(ii) $\frac{1}{1+d} \quad \frac{g(t)}{t} \leq \frac{y(g(t))}{y(t)} \leq 1.$

(ii) can be written as

(iii) $k \leq \frac{y(t-T(t))}{y(t)} \leq 1$ where $0<k<1$, $0 \leq T(t) \leq m$.

The proof of the lemma follows the proof given in (Erbe, 1973) with some modifications.

We are now in a position to give our main results.

Results

1. The following theorem gives necessary and sufficient conditions for oscillation of delay equations of type (1) in the introduction.
(1) in the introduction.

THEOREM I. Suppose that for $uv > 0$,

(i) $\frac{|f(t, u, v)|}{|u|d_1|}$ is nonincreasing in $|u|$ for each v

(ii) $\frac{|f(t, u, v)}{|v|d_2}$ is nonincreasing in $|v|$ for each u

for some d_1, d_2 with $o<d_1+ d_2<1$. Then a necessary and sufficient condition for (1) to be oscillatory is that $\int^{\infty} f(t, t, g(t))\, dt = \infty$.

PROOF. For sufficiency, if $y(t)>0$ is a non oscillatory solution we have

$$\int_{to}^{t} \frac{y''(s)}{(y'(s))^{d_1+d_2}}\, ds + \int_{to}^{t} \frac{f(s, y(s), y(g(s)))}{(y'(s))^{d_1+d_2}} = o.$$

From concavity, since $y'(g(s)) \geq y'(s)$ and $y(s) \leq Ms$ for some $M \geq 1$, we have from the hypothesis and lemma,

$$\frac{f(s, y(s), y(g(s)))}{(y'(s))^{d_1+d_2}} \geq \frac{f(s, y(s), y(g(s)))}{(y'(s))^{d_1}\,(y'(g(s)))^{d_2}}$$

$$\geq k^{d_1+d_2} \frac{f(s, y(s), y(g(s)))}{(y(s))^{d_1}\,(h(g(s))^{d_2}} s^{d_1}\,(g(s))^{d_2}$$

$$\geq k^{d_1+d_2} \frac{f(s, Ms, Mg(s))}{(Ms)^{d_1}\,(M\,g(s))^{d_2}} s^{d_1}\,(g(s))^{d_2}$$

$$\geq A\ f(s, s, g(s)),\ A>0,$$

Since the first integral is finite, we get an immediate contradiction if $\int^{\infty} f(t, t, g(t))\, dt \geq \infty$,

For necessity assume $\int_\infty f(t, t, g(t))\, dt < \infty$. Choose T so that $\int_\infty f(t, t, g(t))\, dt < \frac{1}{2}$ and consider the solution y(t) satisfying $y(t) \equiv 0$, $t \leq T$ and $y'(T) = 1$. The rest of the argument follows as in (Gollwitzer, 1969). This completes the proof of the theorem. Theorem I improves results of Bradley and Gollwitzer (Bradley, 1970; Gollwitzer, 1969). Though Erbe and Wong (Erbe, 1973; Wong, 1972) discuss sublinear equations of the form (1) they do not have results contained in theorem I.

2. For functional delay equations of type (2) in the introduction we have the following result where we will follow the notation of Burkowski and further denote constant functions by bars.

THEOREM II. Suppose $F(t,\psi)$ is nondecreasing with respect to C^+ and C^- and further let $\left|\frac{F(t, \bar{u})}{|u|^d}\right|$ be nonincreasing in $|u|$ for some d, $0<d<1$. Then a necessary and sufficient condition for equation (2) to be oscillatory is that $\int_\infty F(t, t)\, dt = \infty$.

PROOF. As in Theorem I let $y(t)>o$ to be a nonscillatory solution. Then

$$\int_{to}^{t} \frac{y''(s)}{(y'(s))^d}\, ds + \int_{t_o}^{t} \frac{F(s, y_s)}{(y'(s))^d}\, ds = 0.$$

The first term is finite and using Lemma and noting $y(s) \leq Ms, M \geq 1$ we get

$$\frac{F(s, y_s)}{(y'(s))^d} \geq \frac{F(s, \overline{y(s-m)})}{(y'(s-m))^d}$$

$$\geq \frac{A\, F(s, \overline{y(s-m)})}{(y(s-m))^d}\,(s-m)^d, \quad A>0$$

$$\geq A\, \frac{F(s, \overline{M(s-m)})}{[M(s-m)]^d}\,(s-m)^d$$

$$\geq B\, F(s, \overline{s-m}), \quad B>0$$

Further $\geq \frac{F(s, \overline{s-m})}{(s-m)^d} \geq \frac{F(s, \bar{s})}{s^d}$ Hence

$$\frac{F(s, y_s)}{(y'(s))^d} \geq BF(s, \bar{s})\left(\frac{s-m}{s}\right)^d$$

$$\geq C\, F(s, \bar{s})$$

and this leads to a contradiction to $t \to \infty$.

For necessity we follow the same method as in Burkowski and thus complete the proof.

Theorem II includes Theorem I in case g (t) of equation (1) is given by $g(t) = t - T(t)$, $0 \leq T(t) \leq m$. From concavity it follows that for a non oscillatory solution $\lim_{t\to\infty} y'(t) = 0$ or $\lim_{t\to\infty} y' = c < \infty$. With this in

and we can see for equations of the form $y''(t) + F(t, y_t, y'_t) = 0$, Theorem I could be modified as follows: Assume exist $F_1(t, y_t)$, $F_2(t, y_t)$ so that $|F_1(t, \psi)| \leq |F(t,\psi,\phi)| \leq |F_2(t,\psi)|$ for $\psi \in C^+$ or C^-, $\phi \in C^+$ or C^-. It is readily seen that Burkowski's results would be included in Theorem II with appropriate modifications as indicated above.

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