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# TEMPERATURE VARIATION ALONG THE STEM OF A TOTAL HIP JOINT PROSTHESIS

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**ABSTRACT--By approximating the stem of a total hip joint prosthesis as an extended surface, two methods have been used to obtain the temperature variation along it following a temperature rise at the articulating surfaces of the prosthesis (namely, the liner on the acetabular component and the femoral head). The results obtained are within 3% of each other.**

Total hip joint replacement with a prosthesis (Fig. 1) is one of the most common orthopedic surgical procedures, with an estimated 4.5 million such procedures performed worldwide in the period of 1978 to 1988 (Semlitsch, 1989). In spite of the long and wide experience base associated with this procedure, the survival time of such prostheses (defined as time between first insertion and revision due to loosening) in many patients is still regarded as unacceptably low, the average survival time being about 7 years (Pacher et al., 1989). It is realized that such survivorship is a complex phenomenon (Michelson and Riley, 1989). Among the implicating factors in this phenomenon that have been suggested are patient age, weight, sex, and level of activity (Stauffer, 1989), fixation method (Wixson et al., 1991), positioning of the acetabular component (Yoder et al., 1988), cup design (Shelley and Wroblewski, 1988), and size of the head (Morley and Illstrup, 1989). It is felt, nonetheless, that an increase in the survival time of this prosthesis would result from gains in research into and developments in the relevant materials, biomechanical, and heat-transfer aspects.

As far as the materials and biomechanical aspects are concerned, a number of successes have been reported recently. Two examples are fabrication of the head from high-strength ceramics (Boutin et al., 1988) and the use of a telemetric method for recording the hip joint forces experienced during a normal walking cycle (Bergmann et al., 1989).

By contrast, the heat-transfer aspect has, to date, been studied by only a few workers (Davidson and Schwartz, 1987; Davidson et al., 1988*a*; Davidson et al., 1988*b*). There is, thus, a lacuna in the continuum of the field of total-hip-joint-prosthesis research. This matter requires urgent attention given that the survival time of the prosthesis may be reduced by the effects of increases in the rates of creep, oxidation, and wear of the prosthesis (especially when metallic stems and heads are used). Such increases may, in turn, be attributable to the temperature rise (with respect to the surrounding body tissue and fluid) at the articulating cup liner-head interface, and an accompanying high level of frictional heat generation at that interface.

The focus of the present work is the estimation of the temperature variation along the stem following the aforementioned temperature rise. Two methods are utilized in this exercise, with the stem being treated as an extended heat-transfer surface or fin.

## TEMPERATURE RISE AT ARTICULATING SURFACES OF A TOTAL HIP JOINT PROSTHESIS

Recent results (Davidson et al., 1988*b*) from hip joint simulator tests showed that the temperature rise during articulation between the head and the cup liner is a function of the materials of which each component is fabricated, and the maximum applied load on the head, $P_{max}$ (Table 1). In the present work, the patient mass is taken to be 77 kg. The prosthesis type which is considered is one which is in common use. It comprises an ultra-high-molecular-weight polyethylene (UHMWPE) liner on the cup articulating against a head machined from Ti-6Al-4V alloy.

The maximum applied load on the head ($P_{max}$) is put equal to 3000N because results from recent in vivo measurements (Michelson and Reilly, 1989) have indicated that $P_{max}$ is about four times the body weight of the patient. Three other assumptions are employed in the present study. First, it is assumed that the temperature-rise pattern in the Ti-6Al-4V alloy-UHMWPE combination is similar to that pattern reported for the Co-Cr-Mo alloy-UHMWPE combination (Davidson et al., 1988*b*). Second, it is assumed that the in vivo temperature rise in the reconstructed hip joint would be at least 2% higher than the rise measured in the hip simulator tests (Davidson et al., 1988*b*). This is because the lubricant concentration and viscosity in the former case are considerably lower than those used in the latter case. Third, it is assumed that the temperature drop between the articulating surfaces and the bottom of the neck (section WW in Fig. 1) is negligible. Taking all of these factors into consideration, the temperature at section WW, that is $T_o$, is estimated to be 324K.

Two methods are used to obtain the temperature variation along the stem, which is considered fabricated from Ti-6Al-4V alloy (k = 6 W/m/K, p = 4400 kg/m³, and $c_p$ = 610 K/kg/K). Considering the nature of the heat transfer between the stem and the surrounding body tissue and fluid and, following the suggestion of S. Gir (pers. comm.), h is put equal to 40 W/m²/K. The stem dimensions of the prosthesis considered in the present work are given in Fig. 1.

## COMPUTATIONAL METHODS

The two methods employed in obtaining the temperature variation differ from each other on the basis of how the dimensionality of the problem is handled (mathematical nomenclature used is defined in Appendix 1). In the first method (one-dimensional, steady-state triangular fin case), the stem is considered as a truncated triangular fin (Fig. 2). Then, for unit width of the fin, we have the following expressions for the variation of A and P with x:

$$A = wx/L$$
$$P = 2(x - x')\sqrt{1 + (w/2L)^2}\,. \qquad (1)$$

The relevant general one-dimensional steady-state fin equation is

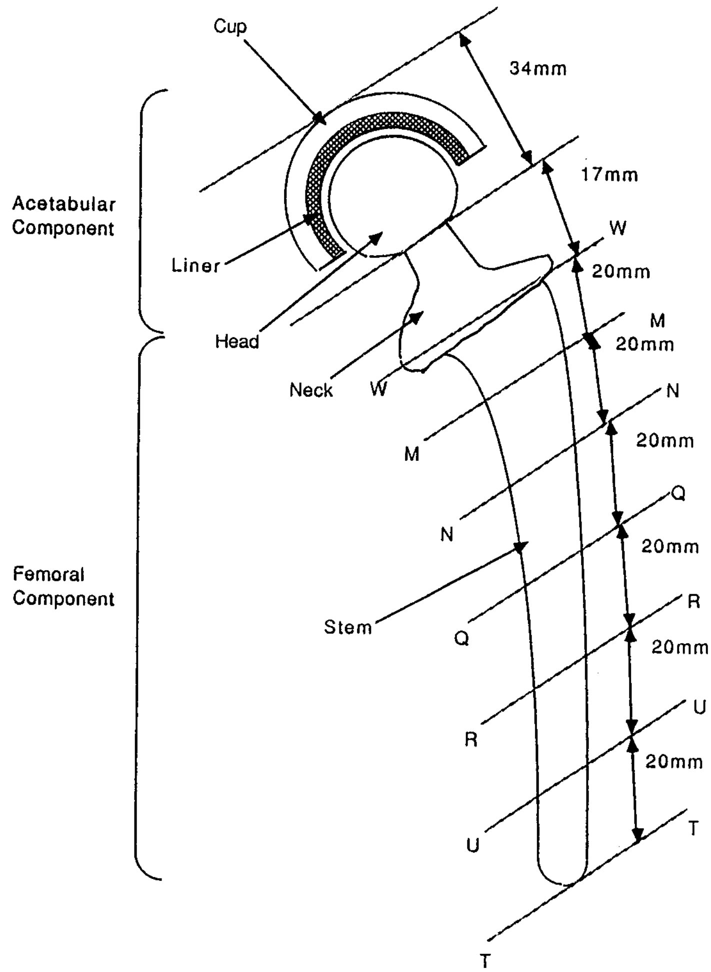


FIG. 1. The components and dimensions of a total hip joint prosthesis.

$$[d^2(T - T_\infty)/dx^2] + [(1/A)(dA/dx)(d[T - T_\infty)/dx] - [(h/k)(1/A)(dP/dx)(T - T_\infty)] = 0. \quad (2)$$

By putting the expressions for A and P from equation (1) into equation (2), we obtain the differential equation

$$[d^2(T - T_\infty)/dx^2] + [(1/x)(d[T - T_\infty]/dx) - [(\beta^2/x)(T - T_\infty)] = 0 \quad (3)$$

where $\beta^2 = (2hL/kw)\sqrt{1 + (w/2L)^2}$.

Equation (3) is a special form of Bessel's zero-order differential equation. Thus, when equation (3) is compared to the generalized Bessel's equations, we obtain the general solution

$$T - T_\infty = C_1 I_0(2\beta x^{0.5}) + C_2 K_0(2\beta x^{0.5}) \quad (4)$$

where the constants $C_1$ and $C_2$ may be determined from the relevant boundary conditions. These conditions are

$$T - T_\infty = T_a \text{ at } x = L \quad (5)$$

and

$$-[kd(T - T_\infty)/dx] = h_t(T - T_\infty) \text{ at } x = x'. \quad (6)$$

From equations (4) and (5), we get

$$T_a = C_1 I_0(2\beta L^{0.5}) + C_2 K_0(2\beta L^{0.5}). \quad (7)$$

From equation (6) and, using the results of the differentiation of Bessel functions, we get

$$-k[C_1\beta x'^{-0.5} I_1(2\beta x'^{0.5}) - C_2\beta x'^{-0.5} K_1(2\beta x'^{0.5})] = h_t[C_1 I_0(2\beta x'^{0.5}) + C_2 K_0(2\beta x'^{0.5})]. \quad (8)$$

Since the stem is short (120 mm), we put $h = h_t$. Then we use equations (7) and (8) to evaluate $C_1$ amd $C_2$. Then, from equation (4), T may be obtained for any specified value of x.

TABLE 1. Equilibrium (30 min) temperature at various combinations of articulating surfaces in a hip joint simulator[1].

| Articulating surfaces[2] | $P_{max}$(N) | Equilibrium temperature (K) |
|---|---|---|
| Polished Co-Cr-Mo alloy on UHMWPE | 2500 | 317 |
| | 5000 | 320 |
| Polished $Al_2O_3$ on UHMWPE | 2500 | 314 |
| | 5000 | 318 |
| Polished $AL_2O_3$ on polished $Al_2O_3$ | 2500 | 312 |
| | 5000 | 315 |

[1]Taken from Davidson et al. (1988*b*).

[2]Lubricant = 5 ml water at 37°C, 1 g/l hyaluronic acid in Ringer's injection fluid at 37°C, or 3.3 g/l hyaluronic acid in Ringer's injection fluid at 37°C.

In the second method (two-dimensional steady-state finite element analysis), the geometry of the stem was represented by a finite-element mesh consisting of 25 linear triangular elements and 21 nodal points (Fig. 3). The temperature variation along the length of the stem was obtained with the aid of a commercially-available two-dimensional steady-state finite element analysis package, Steady-State Heat Transfer Package 101 (ALGOR, Inc., Pittsburgh, Pennsylvania 15238).

## RESULTS AND DISCUSSION

It is seen (Table 2) that, at all sections along the stem, the results of the temperature variation obtained from both methods are within 3% of each other. The finite element results are regarded as being more accurate.

There are two reasons for this contention. First, small rounding-up errors may have entered the computation of the Bessel functions, used in method 1. Second, the finite-element method specifies the geometry in a more accurate way than does method 1.

It has been shown that the temperature variation along the length of the stem of a total hip joint prosthesis may be estimated by considering the stem as an extended surface or fin, with the articulating femoral head-cup liner interface being regarded as the "hot wall." Two levels of sophistication of treatment of the problem produce results that are quite close. This lends credibility to the treatments and the associated assumptions.

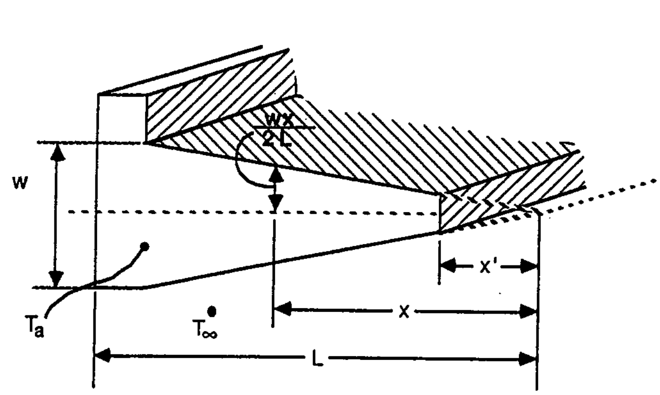


FIG. 2. Geometrical features of a truncated triangular fin.

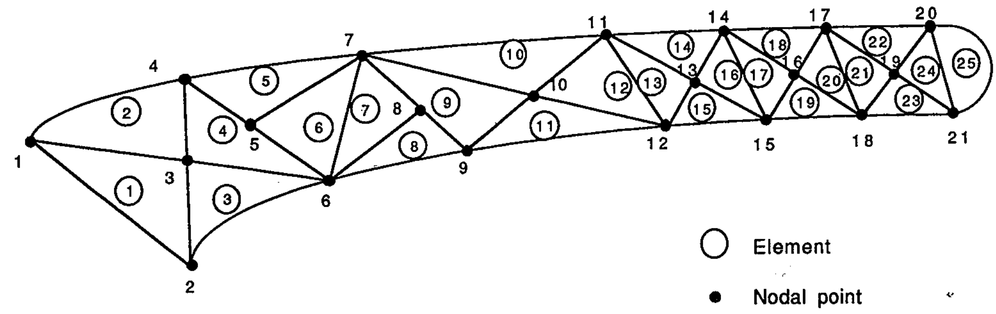


FIG. 3. The 25-element triangular mesh used in the finite-element analysis method.

The clinical significance of the "hot wall effect" and its possible role in the eventual aseptic loosening of the prosthesis (an aspect which provides a perennial challenge to both biomedical and orthopedic workers), while of obvious importance, is outside the scope of the present work and, thus, will not be commented upon here. The drop in temperature along the stem, to a level approximately equal to that of the surrounding body tissue and fluid, underscores its efficiency as a device for dissipating the heat generated at the articulating surfaces.

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TABLE 2. Temperature variation along the stem of a total hip joint prosthesis.

| Section | Distance[1] (mm) | Temperature (K) | |
|---|---|---|---|
| | | Method 1 | Method 2 |
| WW | 0 | 324.0 | 324.0 |
| MM | 20 | 323.1 | 318.1 |
| NN | 40 | 322.1 | 313.5 |
| QQ | 60 | 321.0 | 311.0 |
| RR | 80 | 319.9 | 310.4 |
| UU | 100 | 318.5 | 310.2 |
| TT | 120 | 317.0 | 310.2 |

[1]Distance from section WW.


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## APPENDIX 1

Mathematical nomenclature.

$A$ = cross sectional area of the fin, $m^2$
$c_p$ = specific heat at constant pressure of fin material, J/kg/K
$h$ = mean heat transfer coefficient between the length of the fin and the surrounding body tissue and fluid, $W/m^2/K$
$h_t$ = mean heat transfer coefficient between the tip of the fin and the surrounding body tissue and fluid, $W/m^2/K$
$I_o$ = modified Bessel function of the first kind, order zero
$I_1$ = modified Bessel function fo the first kind, order one
$k$ = thermal conductivity of the fin material, W/m/K
$K_o$ = modified Bessel function of the second kind, order zero
$K_1$ = modified Bessel function of the second kind, order one
$L$ = length of fin, m
$P$ = perimeter of the fin, m
$T$ = temperature at any section along the fin, K
$T_a$ = temperature at the base of the fin (i.e., at section WW), K
$T_\infty$ = temperature of body tissue and fluid medium, K
$w$ = fin coordinate measured normal to heat flow direction, m
$x,x'$ = fin coordinate measured along the horizontal axis, m
$p$ = density of the fin material, $kg/m^3$